

G. S. Mandal's  
 Maharashtra Institute of Technology, Aurangabad  
 (An Autonomous Institute)  
 END SEMESTER EXAMINATION  
First Year B.Tech (All) -April/May 2022

Course Code : BSC101

Course Name : Calculus and Differential Equations

Duration : 2 Hrs

Max. Marks : 50

Date : 5/04/2022

**Instructions :**

- i) All questions are compulsory  
 ii) Assume suitable data wherever necessary and clearly state it  
 iii) Figures to right indicate full marks

Q. 1	Solve/Answer Any Five (Marks: 10)			
	Questions	Marks	CO	BL
a)	State Taylor's Theorem or Series	2	1	1
b)	Test the convergence of the series $\sum_{n=0}^{\infty} \frac{n!}{n^n}$	2	2	2
c)	Write the Condition of Exactness and standard form of its solution	2	3	3
d)	Write the differential equation of R-L Circuit	2	4	3
e)	If $u$ is a function of $x$ and $y$ and $z$ then by Euler's theorem $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = \dots\dots$	2	5	3
f)	If $u = f_1(x, y)$ and $v = f_2(x, y)$ then determinant $\frac{\partial(u, v)}{\partial(x, y)} = \dots\dots$	2	6	3
Q. 2	i) Find the $n^{\text{th}}$ order derivative of $y = \cos^2 x$	4	1	1
	ii) Evaluate $\lim_{x \rightarrow y} \frac{x^y - y^x}{x^x - y^y}$	4	1	
Q. 3	State D'Alamberts ratio test and Examine the convergence of the series $\sum_{n=0}^{\infty} \frac{n^2(n+1)^2}{n!}$	2 6	2	2
Q. 4	i) Solve $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$	4	3	3
	ii) Solve $x \frac{dy}{dx} + y \log y = xe^x y$	4		
Q. 5	A coil having resistance of 15 ohms and an inductance $L$ of 10 henneries is connected to 90 volts power supply. Determine the value of the current after 2 seconds.	8	4	3

	<u>OR</u>			
	<p>A particle of mass 'm' moves under gravity in a medium whose resistance is k times its velocity. If the particle is projected vertically upward direction with initial velocity 'V', Show that the time to reach the maximum height is <math>\frac{m}{k} \log \left[ 1 + \frac{kv}{mg} \right]</math></p>	8	4	3
<b>Q. 6</b>	i) If $u = (1 - 2xy + y^2)^{-1/2}$ find the value of $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y}$	4	5	3
	ii) If $u = \tan^{-1} \left( \frac{y}{x} \right)$ $x = e^t - e^{-t}$ , $y = e^t + e^{-t}$ Find $\frac{du}{dt}$	4	5	
	<u>OR</u>			
	i) Find the Jacobin $\frac{\partial(u, v)}{\partial(x, y)}$ for $u = x + \frac{y^2}{x}$ , $v = \frac{y^2}{x}$	4	6	3
ii) Verify $JJ^* = 1$ for $x = u \cos v$ & $y = u \sin v$	4	6		