

UNIT-1 Matrices

1. Define Rank of matrix

2. Find the rank of the matrix: $\begin{bmatrix} 1 & 2 \\ 7 & 14 \end{bmatrix}$ [Ans: 1]

3. Find the rank of the matrix: $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$ [Ans: 3]

4. Determine the rank of the matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ [Ans: 2]

5. Find the rank of the matrix: $\begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix}$ [Ans: 1]

6. Find the rank of the matrix: $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$ [Ans: 2]

7. Find the rank of the matrix: $\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$ [Ans: 3]

8. Find the rank of the matrix: $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ [Ans: 3]

9. Determine the rank of the matrix: $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ [Ans: 2]

10. Determine the rank of the matrix: $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ [Ans: 2]

11. Find the rank of the matrix: $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$ [Ans: 2]

12. Find the rank of $A + B$, AB and BA if $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 10 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ [Ans: 3,1,1]

1. Define normal form of matrix.

2. Find the rank of the matrix by reducing to its normal form: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ [Ans: 3]

3. Reduce the following matrix into its normal form and hence find its rank

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

[Ans: 3]

4. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ Find two non singular matrices P and Q such that $PAQ = I$

Hence find A^{-1}

$$\text{Ans: } P = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ -2 & 3 & -3 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

5. Find non singular matrices P and Q such that PAQ is a normal form where

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

$$\text{Ans: } P = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -\frac{3}{14} & \frac{1}{28} & \frac{9}{28} \end{bmatrix}, Q = \begin{bmatrix} 1 & -1 & 4 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. Find P and Q such that the normal of $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is PAQ

$$\text{Ans: } P = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \end{bmatrix}, Q = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Find non singular matrices P and Q such that PAQ is a normal form where

$$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

$$\text{Ans: } P = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & -3 & -9 & 7 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8. Solve the following equations

$$x - y + 2z = 3, x + 2y + 3z = 5, 3x - 4y - 5z = -13 \quad [\text{Ans: } x = -1, y = 0, z = 2]$$

9. Discuss the consistency and solve

$$2x + 3y + 4z = 11, x + 5y + 7z = 15, 3x + 11y + 13z = 25$$

$$[\text{Ans: } x = 2, y = -3, z = 4]$$

10. Solve: $3x + 3y + 2z = 1, x + 2y = 4, 10y + 3z = -2, 2x - 3y - z = 5$

$$[\text{Ans: } x = 2, y = 1, z = -4]$$

11. Test the consistency and solve

$$5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$$

$$[\text{Ans: } x = \frac{7}{11}, y = \frac{3}{11}, z = 0]$$

12. Test the consistency and solve

$$x_1 + x_2 + x_3 + x_4 = 0, \quad x_1 + x_2 + x_3 - x_4 = 4$$

$$x_1 + x_2 - x_3 + x_4 = -4, \quad x_1 - x_2 + x_3 + x_4 = 2$$

$$[\text{Ans: } x_1 = 1, x_2 = -1, x_3 = 2, x_4 = -2]$$

13. Solve the following equations

$$2x_1 + x_2 + 2x_3 + x_4 = 6, \quad 6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$

$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1, \quad 2x_1 + 2x_2 - x_3 + x_4 = 10$$

$$[\text{Ans: } x_1 = 2, x_2 = 1, x_3 = -1, x_4 = 3]$$

14. Solve: $x_1 + x_2 - x_3 = 0, 2x_1 - x_2 + x_3 = 3, 4x_1 + 2x_2 - 2x_3 = 2$

$$[\text{Ans: } x_1 = 1, x_2 = k - 1, x_3 = k]$$

15. Solve the following equations

$$x_1 + 2x_2 - x_3 = 1, 3x_1 - 2x_2 + 2x_3 = 2, 7x_1 - 2x_2 + 3x_3 = 5$$

$$[\text{Ans: } x_1 = -\frac{k}{4} + \frac{3}{4}, x_2 = \frac{5k+1}{8}, x_3 = k]$$

16. Test for consistency and solve

$$5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$$

$$[\text{Ans: } x = -\frac{16}{11}k + \frac{7}{11}, y = \frac{3}{11} + \frac{k}{11}, z = k]$$

17. Determine the values of a and b for which the system

$$x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + az = b$$

has (i) no solution (ii) unique solution (iii) infinite number of solutions

18. Find for what values of k the set of equations

$$2x - 3y + 6z - 5t = 3, y - 4z + t = 1, 4x - 5y + 8z - 9t = k$$

has (i) no solution (ii) infinite number of solutions

19. Solve: $x + 2y + 3z = 0, 3x + 4y + 4z = 0, 7x + 10y + 12z = 0$

$$\text{Ans: } x = y = z = 0$$

20. Solve: $x_1 + 3x_2 + 2x_3 = 0, 2x_1 - x_2 + 3x_3 = 0$

$$3x_1 - 5x_2 + 4x_3 = 0, \quad x_1 + 17x_2 + 4x_3 = 0$$

$$[\text{Ans: } x_1 = 11k, x_2 = k, x_3 = -7k]$$

21. Solve: $x + y - 3z + 2w = 0, 2x - y + 2z - 3w = 0$

$$3x - 2y + z - 4w = 0, -4x + y - 3z + w = 0$$

$$\text{Ans: } x = y = z = w = 0$$

22. Solve the system of homogeneous equations

$$4x + 2y + z + 3w = 0, 6x + 3y + 4z + 7w = 0, 2x + y + w = 0$$

$$\text{Ans: } x = k_1, y = -2k_1 - k_2, z = -k_1, w = k_2$$

23. Determine the values of λ such that the system of equations

$$2x + y + 2z = 0, x + y + 3z = 0, 4x + 3y + \lambda z = 0 \text{ have non zero solution}$$

$$\text{Ans: } \lambda = 8$$

24. Find the values of k such that the system of equations

$$x + ky + 3z = 0, 4x + 3y + kz = 0, 2x + y + 2z = 0$$

$$\text{Ans: } k = 0, \frac{9}{2}$$

25. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ find the Eigen values of $3A^3 + 5A^2 - 6A + 2I$

$$\text{Ans: } \lambda = 1, 3, -2 : 4, 110, 10$$

26. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

$$\text{Ans: } \lambda = 1, 2, 3 : X = \begin{bmatrix} k \\ -k \\ 0 \end{bmatrix}, \begin{bmatrix} 2k \\ -k \\ -2k \end{bmatrix}, \begin{bmatrix} k \\ -k \\ -2k \end{bmatrix}$$

27. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$$\text{Ans: } \lambda = -2, 3, 6 : X = \begin{bmatrix} -k \\ 0 \\ k \end{bmatrix}, \begin{bmatrix} k \\ -k \\ k \end{bmatrix}, \begin{bmatrix} k \\ 2k \\ k \end{bmatrix}$$

28. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

$$\text{Ans: } \lambda = 2, 3, 5 : X = \begin{bmatrix} k \\ -k \\ 0 \end{bmatrix}, \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3k \\ 2k \\ k \end{bmatrix}$$

29. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

$$\text{Ans: } \lambda = 1, 2, 2 : X = \begin{bmatrix} k \\ k \\ -k \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

30. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$$\text{Ans: } \lambda = 5, -3, -3 : X = \begin{bmatrix} k \\ 2k \\ -k \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

31. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

$$\text{Ans: } \lambda = 0, 1, 1 : X = \begin{bmatrix} 2k \\ -k \\ 2k \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$$

32. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

$$\text{Ans: } \lambda = 1, 1, 1 : X = \begin{bmatrix} k \\ k \\ k \end{bmatrix}$$

33. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{Ans: } \lambda = 3, 1, 1 : X = \begin{bmatrix} k \\ k \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

34. Define the following terms

- Unique solution
- Trivial solution
- Consistency of equations
- Characteristic equation
- Eigen Values OR Characteristic roots
- Eigen Vectors OR Characteristic vectors

35. State Cayley-Hamilton theorem.

36. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ hence find A^{-1}

$$\text{Ans: } \lambda^3 - \lambda^2 - 4\lambda + 4 = 0, A^{-1} = \frac{1}{4} \begin{bmatrix} 4 & 4 & -4 \\ -2 & -1 & 3 \\ -2 & 1 & 1 \end{bmatrix}$$

37. Find the characteristic equation of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ hence find A^{-1}

$$\text{Ans: } \lambda^3 - 6\lambda^2 + 6\lambda - 11 = 0, A^{-1} = \frac{1}{11} \begin{bmatrix} 5 & -1 & -7 \\ 1 & 3 & 10 \\ -1 & 1 & -2 \end{bmatrix}$$

38. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verified that it is

satisfied by A and hence obtain A^{-1}

Express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ in linear polynomial in A

$$\text{Ans: } \lambda^3 - 6\lambda^2 + 9\lambda - 4I = 0, A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} : 5A - I$$

39. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and verify

Cayley-Hamilton theorem and hence evaluate

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

$$\text{Ans: } \lambda^3 - 5\lambda^2 + 7\lambda - 3I = 0, \quad \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

40. Define linear dependence and linear independence of vectors.
41. Is the system of vectors $X_1 = (2,2,1)^T$, $X_2 = (1,3,1)^T$, $X_3 = (1,2,2)^T$ linearly dependent.
42. Examine the vectors $(1, 2, 4)$, $(2, -1, 3)$, $(0, 1, 2)$, $(-3, 7, 2)$ for linear dependence and find the relation if it exists.
 Ans: $9X_1 - 12X_2 + 5X_3 - 5X_4 = 0$
43. Examine the vectors $[1, 0, 2, 1]$, $[3, 1, 2, 1]$, $[4, 6, 2, -4]$, $[-6, 0, -3, -4]$ for linear dependence and find the relation if it exists.
 Ans: $2X_1 - 6X_2 + X_3 - 2X_4 = 0$

UNIT-II Infinite Series

1. Test the convergence of the series : $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \dots \infty$ [B.tech.Old Nov/Dec 2012]
2. Discuss the convergence of the series: $\sum_{n=1}^{\infty} \left(\frac{n+1}{3n}\right)^n$ [B.tech.Old Nov/Dec 2012]
3. Test the convergence of the series : $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ [B.tech.Old Nov/Dec 2012]
4. Examine the convergence of the sequence: $a_n = \frac{1}{2n}$ [B.tech.Nov/Dec 2012]
5. Test the convergence of the series : $\sum_{n=1}^{\infty} \frac{2n^2+3n}{5+n^5}$ [B.tech.Nov/Dec 2012]
6. If $u_n = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ then u_n is nature [B.tech.Nov/Dec 2012]
7. Test for convergence for series: $\frac{2}{1} + \frac{2.5.8}{1.5.9} + \frac{2.5.8.11}{1.5.9.13} + \dots \infty$ [B.tech.Nov/Dec 2012]
8. Test the convergence for series whose nth term is : $\frac{(n+1)}{n^2} x^n$ [B.tech.Nov/Dec 2012]
9. Discuss the convergence of the series: $\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$ [B.tech.Nov/Dec 2012]
10. Test the convergence of the series: $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$ [B.tech.Nov/Dec 2008]
11. Test the convergence for series whose nth term is : $\frac{n^3}{(n-1)!}$ [B.tech.Nov/Dec 2010]
12. Test the convergence of the series : $\frac{2}{1} + \frac{3}{8} + \frac{4}{27} + \frac{5}{64} + \dots \infty$ [B.tech.Nov/Dec 2010]
13. Test the convergence of the series : $\frac{2}{7} + \frac{2.5}{7.10} + \frac{2.5.8}{7.10.13} + \dots$ [B.tech.Nov/Dec 2010]
14. Discuss the convergence of the series : $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ [B.tech.Nov/Dec 2008]
15. State with reasons the values of x for which the series

- $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \dots$ Converges [B.Tech May/June 2009]
16. Discuss the convergence of the series : $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ [B.Tech May/June 2009]
17. Test the convergence of the series : $\sum_{n=1}^{\infty} \frac{n^4}{n^{6+1}}$ [B.Tech May/June 2009]
18. Test the convergence of the series: $\sum_{n=0}^{\infty} (-1)^n$ [B.tech.May/June 2010]
19. Test for convergence for series: $\frac{1}{1.2.3} + \frac{3}{2.3.5} + \frac{5}{3.4.5} + \dots \infty$ [B.tech.May/June 2010]
20. Using integral test discuss the convergence of the series: $\sum_{n=1}^{\infty} \frac{1}{n^2}$
21. Test the convergence of the series : $\sum_{n=1}^{\infty} \frac{2^n - 1}{2^n}$
22. Discuss the convergence of the series : $\sum_{n=1}^{\infty} \frac{1}{n^n}$
23. Discuss the convergence of the series : $\sum_{n=1}^{\infty} \frac{n^2 + 2^n}{2^n n^2}$ [B.tech.May/June 2010]
24. Test the convergence of the series: $1 + 4 + 7 + 10 + \dots \infty$
25. The ratio test for the positive term series $\sum_{n=1}^{\infty} a_n$ states that..... [F.E.2013]
26. Discuss the convergence of the series : $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ [F.E.2013]

UNIT-III Successive Differentiation

1. Evaluate: $\lim_{x \rightarrow 0} \left[\frac{1}{\text{Log} x} - \frac{x}{x-1} \right]$ [B.Tech.(Old) Nov/Dec 2012]
2. Obtain expansion of e^x in powers of $(x - 2)$. [B.Tech.(Old) Nov/Dec 2012]
3. Using Maclaurin's series expand $\sinh x$ [B.Tech. Nov/Dec 2012]
4. Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \cot^2 x \right)$ [B.Tech. Nov/Dec 2012]
5. Prove that: $e^{x \sin x} = 1 + x^2 + \frac{x^4}{3} + \dots$ [B.Tech. Nov/Dec 2012]
6. Show that: $\sinh^{-1} x = x - \frac{x^3}{6} + \frac{3}{40} x^5 - \dots$ [B.Tech. Nov/Dec 2008]
7. Evaluate: $\lim_{x \rightarrow 0} \frac{1-x^x}{x \cdot \log x}$ [B.Tech. Nov/Dec 2008]
8. Evaluate: $\lim_{x \rightarrow \infty} e^{\frac{\sinh^{-1} x}{\cosh^{-1} x}}$ [B.Tech. Nov/Dec 2010]
9. Prove that: $\text{Log}(x \cot x) = -\frac{x^2}{3} - \frac{7}{90} x^4 - \dots$ [B.Tech. Nov/Dec 2010]
10. Prove that: $\text{Log}(1 + e^x) = \text{Log} 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \dots$ [B.Tech. May/June 2007]
11. Evaluate: $\lim_{x \rightarrow \frac{\pi}{2}} (\text{cosec} x)^{\tan^2 x}$ [B.Tech. May/June 2007]
12. Expand $\tan^{-1} x$ in powers of $(x - 1)$ [B.Tech. May/June 2007]
13. Evaluate: $\lim_{y \rightarrow 1} \frac{y^y - y}{y-1 - \log y}$ [B.Tech. May/June 2009]
14. Prove that: $e^\theta = 1 + \tan \theta + \frac{\tan^2 \theta}{2!} - \frac{\tan^3 \theta}{3!} + \dots$ [B.Tech. May/June 2009]
15. Expand $\sin^{-1} x$ in ascending powers of x [B.Tech. May/June 2009]

16. Prove that: $\cos x \cosh x = 1 - \frac{2^2 x^4}{4!} + \frac{2^4 x^8}{8!} + \dots$ [B.Tech. May/June 2009]
17. Evaluate: $\lim_{x \rightarrow a} \left(2 - \frac{x}{a}\right) \tan\left(\frac{\pi x}{2a}\right)$ [B.Tech. Nov/Dec 2009]
18. Expand e^x in powers of $(x - 1)$ [B.Tech. May/June 2010]
19. Find the value of a and b such that : $\lim_{x \rightarrow 0} (x^{-3} \sin 3x + ax^{-2} + b) = 0$
20. Obtain expansion of $\cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)$ in ascending powers of x [B.Tech. May/June 2010]
21. Expand $(1+x)^{1/x}$ upto the terms containing x^2 [B.Tech. May/June 2008]
22. Evaluate: $\lim_{x \rightarrow 1} (1-x^2) \frac{1}{\text{Log}(1-x)}$ [B.Tech. May/June 2008]
23. Evaluate: $\lim_{x \rightarrow 0} \frac{e^{x \sin x} - \cosh(x\sqrt{2})}{x^4}$ [B.Tech. May/June 2008]
24. Find $\lim_{x \rightarrow a} \log\left(2 - \frac{x}{a}\right) \cot(x-a)$ [B.Tech. Nov/Dec 2009]
25. Evaluate: $\lim_{x \rightarrow 0} \frac{x e^x - \text{Log}(1+x)}{x^2}$ [B.Tech. May/June 2012]
26. Expand $\log_e x$ in powers of $(x - 1)$ [B.Tech. May/June 2012]
27. Evaluate: $\lim_{x \rightarrow 0} \frac{\text{Log} x}{\cot x}$ [B.Tech. May/June 2012]
28. Using Maclaurin's series expand $\sin x$ in ascending powers of x
29. Expand $e^{\sin x}$ upto the terms containing x^4 [B.Tech. May/June 2012]
30. Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ [F.E. May/June 2013]
31. Prove that: $\text{Log}(1 + \sin x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \dots$ [F.E. May/June 2013]
32. Evaluate: $\lim_{x \rightarrow 0} \frac{\cosh x - \cos x}{x \sin x}$ [F.E. May/June 2013]
33. Prove that:

$$\text{Log} x = \text{Log} 2 + \frac{x-2}{2} - \frac{(x-2)^2}{8} + \frac{(x-2)^3}{24} - \dots$$
 [F.E. Nov/Dec 2009]
34. Prove that: $\sec^2 x = 1 + x^2 + \frac{2}{3} x^4 + \dots$ [F.E. Nov/Dec 2009]
35. Prove that: $\lim_{x \rightarrow 0} \left[\cot x - \frac{1}{x} \right] = 0$ [F.E. Nov/Dec 2009]
36. Prove that: $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{1/x} = 1$ [F.E. Nov/Dec 2009]
37. Prove that: $\text{Log} x = (x - 1) - \frac{1}{2} (x - 1)^2 + \frac{1}{3} (x - 1)^3 - \dots$ [F.E. May/June 2012]
38. Evaluate: $\lim_{x \rightarrow 0} (1 + \tan x)^{\cot x}$ [F.E. May/June 2012]
39. Use Taylor's theorem to find $\sqrt{25.15}$ [F.E. Nov/Dec 2008]
40. Find: $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e + \frac{e^x}{2}}{x^2}$ [F.E. Nov/Dec 2008]
41. Prove that: $\text{Log}(1 + e^x) = \text{Log} 2 + \frac{x}{2} + \frac{x^2}{2} - \dots$ [F.E. Nov/Dec 2008]
42. Prove that: $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2 \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \right]$ [F.E. May/June 2008]

43. Evaluate: $\lim_{x \rightarrow 0} \frac{\sin x \sin^{-1} x}{x^2}$ [F.E. May/June 2008]
44. Evaluate: $\lim_{x \rightarrow 0} (\cos^2 x)^{1/x^2}$ [F.E. May/June 2008]
45. Evaluate: $\lim_{x \rightarrow 1} (1-x) \tan\left(\frac{\pi x}{2}\right)$ [F.E. Nov/Dec 2007]
46. Expand $\cos^2 x$ in ascending powers of x [F.E. Nov/Dec 2007]
47. Prove that: $\text{Log}(\sec x) = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{2}{90}x^6 + \dots$ [F.E. Nov/Dec 2007]
48. Evaluate: $\lim_{x \rightarrow 0} x \text{Log} x$ [B.Tech. May/June 2014]
49. Evaluate: $\lim_{x \rightarrow 1} (1-x^2)^{\frac{1}{\text{Log}(1-x)}}$ [B.Tech. May/June 2014]
50. Find Taylor's series expansion for $\text{Log} \cos x$ about the point $\frac{\pi}{3}$ [B.Tech. May/June 2014]

UNIT-IV Complex Number

1. Prove that:
 $(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i \sin\theta)^n = 2^{n+1} \cos^n\left(\frac{\theta}{2}\right) \cos\left(\frac{n\theta}{2}\right)$
 (B.tech. Nov/Dec 2012)
2. Find the locus of $|z - 2i| = 3$ (B.tech. Nov/Dec 2012)
3. If n is a positive integer, then prove that :
 $(1 + i)^n + (1 - i)^n = (\sqrt{2})^{n+2} \cos\left(\frac{n\pi}{4}\right)$ (B.tech. Nov/Dec 2012)
4. Solve the equation: $x^9 - x^5 + x^4 - 1 = 0$ (B.tech. Nov/Dec 2012)
5. Find what 'Amplitude of $\frac{z-1}{z+1}$ is $\frac{\pi}{4}$ ' represent. (B.tech. Nov/Dec 2008)
6. Show that the roots of $(x+1)^6 + (x-1)^6 = 0$ are given by: $-i \cot\left[\frac{(2m+1)\pi}{12}\right]$
 where $0 \leq m \leq 5$ (B.tech. Nov/Dec 2008)
7. If $u = \frac{z+i}{z+2}$ and $z = x + iy$ then show that, locus of (x, y) is a circle if u is purely imaginary find the center and radius of the circle. (B.tech. Nov/Dec 2010)
8. If $z = x + iy$ where x and y are real, show that when $\frac{z+i}{z+2}$ is real, the locus of the point (x, y) is a straight line. (B.tech. Nov/Dec 2007)
9. If $z = x + iy$ and x, y are real, if $\frac{z+i}{z+2}$ is real, then find the locus of (x, y)
 (B.tech. Nov/Dec 2009,12)
10. If $x_n + iy_n = (1 + i\sqrt{3})^n$, then prove that $x_{n-1}y_n - x_n y_{n-1} = 4^{n-1}\sqrt{3}$
 (B.tech. May/June 2009,12)
11. Determine the region in the z -plane represented by:
 (i) $R\left(\frac{1}{z}\right) < \frac{1}{2}$ (ii) $I(z) > 1$ (B.tech. Nov/Dec 2009)
12. Determine the region in the z -plane represented by:
 (i) $R(z^2) \leq 1$ (ii) $\frac{\pi}{3} < \text{amp}(z) < \frac{\pi}{2}$ (B.tech. May/June 2010)

13. Solve the equation: $x^8 + x^5 + x^3 + 1 = 0$ (B.tech. Nov/Dec 2009)
14. Solve the equation: $x^7 + x^4 + x^3 + 1 = 0$ (B.tech. May/June 2010,2012)
15. Using De-moivre's theorem solve the equation:
 $(x - 1)^5 + x^5 = 0$ (B.tech. Nov/Dec 2008)
16. Solve the equation: $x^7 - x^4 + x^3 - 1 = 0$ (B.tech. Nov/Dec 2007)
17. If $|z_1 + z_2| = |z_1 - z_2|$ prove that: $\text{amp}\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ where z_1 and z_2 are any two complex numbers. (B.tech. Nov/Dec 2007)
18. Determine the region in the z-plane represented by $1 < |z + 2i|$ (B.tech. Apr/May 2012)
19. Find cube roots of unity. (B.tech. Apr/May 2012)
20. For a complex number $= 1 + i$, $|z| = \dots\dots$ and $\text{amp}(z) = \dots\dots$ (F.E.May/June 2013)
21. If z_1 and z_2 be two complex numbers such that $|z_1 + z_2| = |z_1 - z_2|$ prove that the difference of their amplitude is $\frac{\pi}{2}$. (F.E.May/June 2013,09)
22. If n is a positive integer, then prove that :
 $(1 + i)^n + (1 - i)^n = (\sqrt{2})^{n+2} \cos\left(\frac{n\pi}{4}\right)$ (F.E.May/June 2013)
23. Prove that: $(x + iy)^{m/n} + (x - iy)^{m/n} = 2(x^2 + y^2)^{m/2n} \cos\left[\frac{m}{n} \tan^{-1}\left(\frac{y}{x}\right)\right]$ (F.E.Nov/Dec 2009)
24. If $\arg(z + 1) = \frac{\pi}{6}$ and $\arg(z - 1) = \frac{2\pi}{3}$ find z. (F.E.April/May 2012)
25. Find the roots of the polynomial:
 $x^4 - x^3 + x^2 - x + 1 = 0$ (F.E.April/May 2012)
26. Prove that: $\left[\frac{1 + \sin\alpha + i\cos\alpha}{1 + \sin\alpha - i\cos\alpha}\right]^n = \cos\left[\frac{n\pi}{2} - n\alpha\right] + i\sin\left[\frac{n\pi}{2} - n\alpha\right]$ (F.E.Nov/Dec 2008)
27. Find the different values of $(1 + i)^{1/3}$ (F.E.May/June 2008)
28. Find the locus represented by: $|z + 2 - i| = 4$ (F.E.May/June 2008)
29. Solve by using complex number:
 $x^{10} + 11x^5 + 10 = 0$ (F.E.Nov/Dec 2007)
30. Prove that the sum of n^{th} roots of unity is zero. (F.E.Nov/Dec 2007)
31. Prove that: $\coth^{-1}x = \frac{1}{2} \text{Log} \frac{x+1}{x-1}$, $|x| > 1$ [B.tech.Old Nov/Dec 2012]
32. If $\cosh(u + iv) = x + iy$ prove that: $\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1$, $\frac{x^2}{\cos^2 u} - \frac{y^2}{\sin^2 u} = 1$ [B.tech.Old Nov/Dec 2012]
33. If $i^{\alpha+i\beta} = \alpha + i\beta$ prove that: $\alpha^2 + \beta^2 = e^{-\pi\beta(4n+1)}$ [B.tech.Old Nov/Dec 2012]
34. If $\tan(\theta + i\Phi) = \tan\alpha + i \sec\alpha$ prove that:
 $e^{2\phi} = \cot \frac{\alpha}{2}$, $2\theta = n\pi + \frac{\pi}{2} + \alpha$ [B.tech. Nov/Dec 2012]
35. Prove that: $\text{Log} \left[\frac{\cos(x-iy)}{\cos(x+iy)} \right] = 2i \tan^{-1}(\tan x \tanh y)$ [B.tech. Nov/Dec 2012]
36. Prove that: $\tanh^{-1}x = \sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ [B.tech. Nov/Dec 2012]
37. Prove that: $\text{sech}^{-1}[\sin\alpha] = \text{logcot}\left(\frac{\alpha}{2}\right)$ [B.tech. May/June 2008]

38. Prove that: $\text{Log} \left[\frac{(a-b)+i(a+b)}{(a+b)+i(a-b)} \right] = i \left[2n\pi + \tan^{-1} \frac{2ab}{a^2-b^2} \right]$ [B.tech. May/June 2008]
39. Prove that: $\sinh^{-1}x = \text{Log}(x + \sqrt{x^2 + 1})$ [B.tech.May/June 2008]
40. If $\sin(\theta + i\phi) = e^{i\alpha}$ prove that: $\cos^2\theta = \pm \sin\alpha$ [B.tech. May/June 2007]
41. Separate into real and imaginary parts: $\sec(x + iy)$ [B.tech. May/June 2007]
42. If $\tan(A + iB) = \alpha + i\beta$ show that: $\frac{1-(\alpha^2+\beta^2)}{1+(\alpha^2+\beta^2)} = \frac{\cos 2A}{\cosh 2B}$ [B.tech. May/June 2009]
43. Find principal value of $\frac{(a+ib)^{p+iq}}{(a-ib)^{p-iq}}$ [B.tech. May/June 2009]
44. If $x = 2\sin\alpha \cosh\beta$, $y = 2\cos\alpha \sinh\beta$ show that:
- (i) $\text{cosec}(\alpha - i\beta) + \text{cosec}(\alpha + i\beta) = \frac{4x}{x^2+y^2}$
- (ii) $\text{cosec}(\alpha - i\beta) - \text{cosec}(\alpha + i\beta) = \frac{i4y}{x^2+y^2}$ [B.tech. Nov/Dec 2009]
45. Prove that: $i^a = \cos \left[\left(2m + \frac{1}{2} \right) \pi \alpha \right] + i \sin \left[\left(2m + \frac{1}{2} \right) \pi \alpha \right]$ [B.tech. Nov/Dec 2009]
46. Prove that: $\text{Logtan} \left(\frac{\pi}{4} + \frac{x}{2} i \right) = i \tan^{-1}(\sinh x)$ [B.tech. May/June 2010]
47. Prove that :
- (i) $\tanh^{-1}x = \sinh^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right)$
- (ii) $\sinh^{-1}x = \frac{1}{2} \text{cosech}^{-1} \frac{1}{2x\sqrt{1-x^2}}$ [B.tech.Nov/Dec 2008]
48. Prove that: $\tan \left[i \log \left(\frac{a-ib}{a+ib} \right) \right] = \frac{2ab}{a^2-b^2}$ [B.tech.Nov/Dec 2008]
49. Separate $\tan^{-1}(x + iy)$ into real and imaginary parts. [B.tech.Nov/Dec 2008]
50. Prove that: $\tanh^{-1}x = \frac{1}{2} \text{Log} \frac{1+x}{1-x}$ [B.tech.Nov/Dec 2007]
51. If $\tan(\alpha + i\beta) = x + iy$ show that: $x^2 + y^2 + 2x \cot 2\alpha = 1$ [B.tech. May/June 2012]
52. Prove that: $i^i = e^{-(2n+\frac{1}{2})\pi}$ [B.tech. May/June 2012]
53. Separate into real and imaginary parts of $\cos^{-1} \left(\frac{3i}{4} \right)$ [B.tech. May/June 2012]
54. Separate into real and imaginary parts $\sin(x - iy)$ [B.tech. Nov/Dec 2012]
55. If $\cosh(\alpha + i\beta) = a + ib$ then $a = \dots\dots$ and $b = \dots\dots$ [F.E. May/June 2013]
56. Show that: $64 \sin^7\theta = 35\sin\theta - 21 \sin 3\theta + 7 \sin 5\theta - \sin 7\theta$ [F.E. May/June 2013]
57. Prove that: $\text{Log} \left(\frac{1}{1-e^{i\theta}} \right) = \text{Log} \left(\frac{1}{2} \text{cosec} \frac{\theta}{2} \right) + i \left(\frac{\pi}{2} - \frac{\theta}{2} \right)$ [F.E. May/June 2013]
58. Prove that: $\sin 7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64 \sin^7\theta$ [F.E.Nov/Dec 2009]
59. If $\sin(\alpha + i\beta) = x + iy$ prove that: $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1$ [F.E.Nov/Dec 2009]
60. Expand $\cos 3\theta$ using De-Moivre's theorem. [F.E.April/May 2012]
61. Find the value of a and b if $a + ib = i^{(1+i)}$ [F.E.April/May 2012]
62. Expand $\sin^5\theta$ in a series of sines multiples of θ . [F.E.April/May 2012]
63. Prove that the real part of the principal values of $i^{\log(1+i)}$ is $e^{-\pi^2/8} \cos \left(\frac{\pi}{4} \log 2 \right)$
64. Show that: $\sqrt[n]{x+iy} + \sqrt[n]{x-iy}$ has n real values and find those of

$$\sqrt[3]{1+i\sqrt{3}} + \sqrt[3]{1-i\sqrt{3}} \quad [\text{F.E.Nov/Dec 2008}]$$

65. Prove that:

$$128[1 - \cos^2\theta]^4 = \cos 8\theta - 8 \cos 6\theta + 28 \cos 4\theta - 56 \cos 2\theta + 35 \quad [\text{F.E.Nov/Dec 2008}]$$

66. If $\cos(x + iy) = e^{i\alpha}$ show that: $\cosh 2y + \cos 2x = 2$ [F.E. May/June 2008]

67. Simplify : $\left[\frac{1 + \cos(\frac{\pi}{9}) + i \sin(\frac{\pi}{9})}{1 + \cos(\frac{\pi}{9}) - i \sin(\frac{\pi}{9})} \right]^{18}$ [F.E.Nov/Dec 2007]

68. Expand $\sin^5\theta \cos^3\theta$ in a series of sines multiples of θ . [F.E.Nov/Dec 2007]

UNIT-V Partial Differentiation

1. If $x = \frac{\cos\theta}{u}$, $y = -\frac{\sin\theta}{u}$ and $z = f(x, y)$ show that :

$$\left(\frac{\partial x}{\partial u}\right)_{\theta} \left(\frac{\partial u}{\partial x}\right)_y = \cos^2\theta \quad [\text{B.Tech.(Old)Nov/Dec 2012}]$$

2. If $z(x + y) = x^2 + y^2$ show that : $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4 \left[1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]$

3. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u (1 - 4 \sin^2 u) \quad [\text{B.Tech.(Old)Nov/Dec 2012}]$$

4. If $x = uv$, $y = \frac{u}{v}$, $z = f(x, y)$ prove that $\frac{\partial z}{\partial x} = \frac{1}{2v} \frac{\partial z}{\partial u} + \frac{1}{2u} \frac{\partial z}{\partial v}$ [B.Tech.Nov/Dec 2012]

5. If $z = x^2 + y^2$, $x = at \sin t$, $y = at \cos t$ find $\frac{dz}{dt}$ [B.Tech.(Old)Nov/Dec 2012]

6. If $\theta = t^n e^{-\frac{r^2}{4t}}$ find $\frac{\partial \theta}{\partial t}$. [B.Tech.Nov/Dec 2012]

7. Find $\frac{dy}{dx}$ if $y^x + x^y = (x + y)^{x+y}$ [B.Tech.Nov/Dec 2012]

8. If $u = ax + by$, $v = bx - ay$ find the value of $\left(\frac{\partial x}{\partial u}\right)_v \left(\frac{\partial y}{\partial v}\right)_u$

9. If $\text{Log}(\sin u) = \frac{(x^2+y^2)^{3/2}}{x+y}$ find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ [B.Tech.Nov/Dec 2012]

10. If $x = e^{\theta+i\phi}$, $y = e^{\theta-i\phi}$ prove that $J J' = 1$ [B.Tech.Nov/Dec 2012]

11. If $z = f(x, y)$, $x = u \cos \alpha - v \sin \alpha$, $y = u \sin \alpha + v \cos \alpha$ prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \quad [\text{B.Tech.Nov/Dec 2008}]$$

12. If $2x = r[e^{\theta} + e^{-\theta}]$, $2y = r[e^{\theta} - e^{-\theta}]$ then prove that $\left(\frac{\partial x}{\partial r}\right)_{\theta} = \left(\frac{\partial r}{\partial x}\right)_y$

13. Find $\frac{du}{dx}$, given that: $u = x \text{Log} xy$ and $x^3 + y^3 = -3xy$ [B.Tech.Nov/Dec 2008]

14. If $x^x y^y z^z = c$ when $x = y = z$ Prove that $\frac{\partial^2 z}{\partial x \partial y} = -[x (\text{Log} x)]^{-1}$

If $z = f(x, y)$ where $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ then prove that :

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} \quad [\text{B.Tech.Nov/Dec 2008}]$$

15. If $u = \sin^{-1} \left[\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \right]$ show that: $u_x + \frac{y}{x} u_y = 0$ [B.Tech.Nov/Dec 2008]

16. If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$ find $\frac{\partial(u,v)}{\partial(x,y)}$ [B.Tech.Nov/Dec 2008]

17. If $\tan^2 \theta = \frac{y}{x}$, show that $\left(\frac{\partial \theta}{\partial x}\right)_y \left(\frac{\partial \theta}{\partial y}\right)_x = \frac{-1}{4(x+y)^2}$ [B.Tech.Nov/Dec 2010]

18. If $u = \operatorname{cosec}^{-1} \sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$ prove that :

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{\tan u}{12} \left[\frac{13}{12} + \frac{\tan^2 u}{12} \right] \quad [\text{B.Tech.Nov/Dec 2010}]$$

19. If $u(x+y) = x^2 + y^2$ show that : $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4 \left[1 - \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) \right]$

20. If $\theta = t^n e^{-\frac{r^2}{4t}}$ then find the value of n which will make

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t} \quad [\text{B.Tech.Nov/Dec 2010}]$$

21. If $u = \operatorname{Log}(x^3 + y^3 - x^2y - xy^2)$ prove that:

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -3 \quad [\text{B.Tech.May/June 2007}]$$

22. Find $\frac{dz}{dt}$ when $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$. Verify by direct substitution.

23. Find the stationary values of : $x^3y^2(1-x-y)$ [B.Tech.Nov/Dec 2009]

24. Find $\frac{du}{dx}$ if $u = \tan^{-1} \left(\frac{x}{y} \right)$, x and y are related by $x^2 + y^2 = a^2$

25. If $u = \sin(\sqrt{x} + \sqrt{y})$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} (\sqrt{x} + \sqrt{y}) \cos(\sqrt{x} + \sqrt{y})$

26. If $r^2 = x^2 + y^2 + z^2$ and $u = r^m$ prove that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2} \quad [\text{B.Tech.Nov/Dec 2009}]$$

27. If $x = r \cosh \theta$, $y = r \sinh \theta$, $z = f(x, y)$ prove that:

$$(x-y) \left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right) = r \frac{\partial z}{\partial r} - \frac{\partial z}{\partial \theta} \quad [\text{B.Tech.Nov/Dec 2009}]$$

28. If $u = \sin^{-1} \left(\frac{x^{1/4} - y^{1/4}}{x^{1/5} + y^{1/5}} \right)$ prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{400} [\tan^2 u - 19] \quad [\text{B.Tech.May/June 2010}]$$

29. Find $\frac{dz}{dt}$ if $z = xy^3 + x^3y$, $x = at^2$, $y = 2at$ [B.Tech.May/June 2010]

30. If $f = x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{x}{y} \right)$ show that :

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2} \quad [\text{B.Tech.May/June 2010}]$$

31. If $z = f(u, v)$ and $u = e^\theta \cos \phi$, $v = e^\theta \sin \phi$ show that

$$\frac{\partial^2 z}{\partial \theta^2} + \frac{\partial^2 z}{\partial \phi^2} = (u^2 + v^2) \left[\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right] \quad [\text{B.Tech.May/June 2010}]$$

32. If $u = a \cosh x \cos y$, $v = a \sinh x \sin y$ show that

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{a^2}{2} [\cosh 2x - \cos 2y] \quad [\text{B.Tech.Nov/Dec 2008}]$$

33. If $x^2 = au + bv$, $y^2 = au - bv$ then prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u = \frac{1}{2}$

34. If $u = \sin\left(\frac{x}{y}\right)$ where $x = e^t, y = t^2$ find $\frac{du}{dt}$ [B.Tech.Nov/Dec 2007]
35. Find the value of n so that $u = r^n [3\cos^2\theta - 1]$ satisfies the partial differential equation

$$\frac{\partial}{\partial r}\left[r^2 \frac{\partial u}{\partial r}\right] + \frac{1}{\sin\theta} \frac{\partial}{\partial \theta}\left[\sin\theta \frac{\partial u}{\partial \theta}\right] = 0$$
 [B.Tech.Nov/Dec 2007]
36. If $u = \sinh^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$
37. If $z = e^{xy^2}, x = t \cos t, y = t \sin t$ find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$ [B.Tech.Nov/Dec 2007]
38. If $u = f_1(x, y)$ where $x = f_2(t), y = f_3(t)$ then $\frac{du}{dt} = \dots$ [F.E.2013]
39. If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right)$ then $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \dots$ [F.E.2013]
40. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$
41. If $u = 2xy, v = x^2 - y^2, x = r \cos\theta, y = r \sin\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$ [F.E.2013]
42. If $u = \sin^{-1}(x^2 + y^2)$ find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$
43. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that: $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$
44. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ [F.E.2009]
45. If $z = f(x, y), u = lx + my, v = ly - mx$ show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2) \left[\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right]$$
 [F.E.2009]
46. If $u = x^2 y^2, x = 2at, y = at^3$ find $\frac{du}{dt}$ [F.E.2012]
47. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \dots$ [F.E.2012]
48. $f(x, y)$ has a maximum value at (a,b) if [F.E.2012]
49. If $u = \frac{y}{z} + \frac{z}{x}$ show that: $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ [F.E.2012]
50. If $u + x + y + z, uv = y + z, uvw = z$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$ [F.E.2012]
51. If $x = \frac{vw}{u}, y = \frac{wu}{v}, z = \frac{uv}{w}$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = 4$ [F.E.2008]
52. If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ and $u = r \sin\theta \cos\phi, v = r \sin\theta \sin\phi, w = r \cos\theta$ find the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ [F.E.2008]
53. If $y = x \cos u$, then find the value of
 (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$
 (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ [F.E.2007]
54. If $ux + vy = 0, \frac{u}{x} + \frac{v}{y} = 1$ prove that $\left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial v}{\partial y}\right)_x = \frac{x^2 + y^2}{y^2 - x^2}$ [F.E.2007]

UNIT-VI Maxima and Minima

1. State necessary condition for existence of maxima and minima for $u = f(x, y)$.
2. $f(x, y)$ has minimum value at (a,b) if..... [F.E.2013]
3. The stationary points of the function: $f(x, y) = 3x^2 - y^2 + x^3$ will be.....
4. Find stationary values of: $x^3 + y^3 - 3axy, a > 0$
[B.Tech.Nov/Dec2008]
5. Examine for minimum and maximum values of: $x^2 + y^2 + (10 - x - y)^2$
6. Find the stationary values of: $x^3y^2(1 - x - y)$ [B.Tech.Nov/Dec 2009]
7. Find maxima and minima of: $xy + \frac{a^3}{x} + \frac{a^3}{y}, a > 0.$ [B.Tech.May/June 2010]
8. Discuss maxima and minima of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$
9. Examine the minimum and maximum value of: $\sin x + \sin y + \sin(x + y)$.
10. Discuss the maxima and minima of: $x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$ [F.E.2008]
11. If $x = u(1 - v), y = uv$ then prove that: $J J' = 1.$
[B.Tech.(Old)Nov/Dec 2012]
12. If $x = e^{\theta+i\phi}, y = e^{\theta-i\phi}$ prove that $J J' = 1$ [B.Tech.Nov/Dec 2012]
13. If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$ prove that: $J J' = 1$
14. Show that: $\frac{\partial(r,s,t)}{\partial(x,y,z)} = x^2y$ use the following information:
 $x = r + s + t, xy = s + t, xyz = t$ [B.Tech.Nov/Dec 2010]
15. If $x + y = 2e^{\theta} \cos\phi, x - y = 2ie^{\theta} \sin\phi$ prove that: $J J' = 1$ [B.Tech.Nov/Dec 2009]
16. If $x = uv, y = \frac{u}{v}$ prove that: $J J' = 1$ [B.Tech.Nov/Dec 2009]
17. If $x = e^{\phi} \sec\theta, y = e^{\phi} \tan\theta$ then prove that: $J J' = 1$ [B.Tech.May/June 2010]
18. If $x = r \cos\theta, y = r \sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} = \dots$ [F.E.2013]
19. If $x = e^{\alpha} \cos\beta, y = e^{\alpha} \sin\beta$ then show that $J J' = 1$ [F.E.2008]
20. If $x = \frac{vw}{u}, y = \frac{wu}{v}, z = \frac{uv}{w}$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = 4$ [F.E.2008]
21. If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ and $u = r \sin\theta \cos\phi, v = r \sin\theta \sin\phi,$
 $w = r \cos\theta$ find the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ [F.E.2008]
22. If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ find the Jacobian of the given transformation.

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