CODE NO. : P-1010-2013

(Revised Course)

FACULTY OF ENGONEERING AND TECHNOLOGY

S.Y.B.Tech.(All) EXAMINATION

NOVEMBER/DECEMBER, 2013

ENGINEERING MATHEMATICS-IV

Time-Three Hours Maximum Marks-80

''Please check whether you have got the right question paper.''

N.B. :- (i) Question No.1 and Question No.6 are compulsory.

(ii) From section A solve any two section questions from the remaining Q.No.2,3,4 and 5

(iii) From section B solve any two section questions from the remaining Q.No.7,8,9 and 10

(iv) Figures to the right indicate full marks.

(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks) 10

(a) If $f'(z) = cosx\ cosy + i\ sinx\ siny$ then $f(z) =$ ……….

(b) Find Residue of $f(z) = tanz$ at $z = \frac{\pi}{2}$

(c) Evaluate: $\int_C \frac{cosz}{z}\ dz$, $C: |z| = 1$

(d) Evaluate: $\int_0^{\pi i} sin^2 z\ dz$

(e) A bilinear transformation that maps three distinct points z_1, z_2, z_3 onto three points w_1, w_2, w_3 is given by the formula ……….

(f) State Gauss-divergence theorem

(g) If $\overline{F}.d\overline{r} = d(x^2y) + d(xz^3)$ then find $\int_{(1,2,1)}^{(3,0,1)} \overline{F}\,.\,d\overline{r}$.

(h) Define conservative field.

2. (a) Find the analytic function $f(z) = u + iv$ given that

$v = e^x(x\,siny + y\,cosy)$ 5

(b) Find the work done by a force $\overline{F} = yzi + xzj + xyk$ along

$x = acost\,, y = a\,sint\,, z = at$ from $t = 0$ to $t = \frac{\pi}{4}$ 5

(c) Evaluate: $\int_0^{2+i} (\overline{z})^2\; dz$

(i) along the line $y = \frac{x}{2}$

(ii) on the real axis from 0 to 2 and then vertically to $2 + i$ 5

3. (a) Show that: $u = e^x(x\,cosy - y\,siny)$ is harmonic function. Find its harmonic conjugate. 5

(b) Use Cauchy's integral formula to evaluate: $\int_C \frac{logz}{(z-1)^3}\, dz$ where C: $|z - 1| = \frac{1}{2}$ 5

(c) Apply Green's theorem to evaluate: $\int_C \overline{F}\, d\overline{r}$ where $\overline{F} = 2y^2 i + 3xj$ over $y = x^2$ and $y = x$ 5

4. (a) Evaluate : $\int_C \frac{z}{(z-1)(z-2)^2}\, dz$ where C : $|z - 2| = \frac{1}{2}$ by Cauchy's Residue theorem. 5

(b) Find the image of the infinite strip :

(i) $\frac{1}{4} < y < \frac{1}{2}$ (ii) $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ 5

(c) Evaluate: $\iint_S (\nabla \times \overline{F}).\, ds$ where $\overline{F} = (x^3 - y^3)i - xyzj + y^3k$ and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$ 5

5. (a) Expand: $\frac{1}{(z-1)(z-2)}$ for $1 < |z - 3| < 2$ in Laurent series. 5

(b) Evaluate: $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+4)(x^2+9)} dx$ by using residues. 5

(c) Evaluate: $\iint_S (x^3 i + y^3 j + z^3 k) ds$ where $x^2 + y^2 + z^2 = 16$. 5

Section B

6. Solve any five (Each for two marks) 10

(a) Find $Z[\left(\frac{1}{3}\right)^{n} \sin\left(\frac{n\pi}{2}\right)]$

(b) Find $Z[sin^3 n]$

(c) Find $Z^{-1}\left[\frac{z}{(z-1)(z-3)}\right]$

(d) Equation: $x - e^{-x} = 0$ is required to be solved by Newton-Raphson method with an initial guess $x_0 = 1$. Then find first approximation of the root (i.e. x_1) correct to 3 decimal places.

(e) Find the values of x, y, z in the first iteration for the system of equations

$10x + y + z = 18.141$

$x + y + 10z = 38.139$

$x + 10y + z = 28.140$ to be solved by Gauss-Seidel method

(f) Find $f(2)$ for the following data:

x	$f(x)$
1	2
3	3
5	7

(g) If $u = c\, e^{k\left(x+\frac{y}{4}\right)}$ is the solution of $\frac{\partial u}{\partial x} = 4\,\frac{\partial u}{\partial y}$, $u(0,y) = 8e^{-3y}$ find c and k.

(h) Write suitable solution of one dimensional heat flow equation: $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

7. (a) Solve: $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u$, $u(x,0) = 3e^{-5x} + 2e^{-3x}$ by method of separation of variables. 5

(b) Solve by Gauss-Seidel method: 5

$54x + y + z = 110$, $2x + 15y + 6z = 72$, $-x + 6y + 27z = 85$ 5

(c) Find by Taylor's series method the value of y at $x = 0.1$ correct to three decimal places: $\frac{dy}{dx} = x^2 y - 1$, $y(0) = 1$ 5

8. (a) Using Newton-Raphson method, find a root of the equation (correct to 3 decimal):

$xe^x - 2 = 0$ 5

(b) Solve: $\frac{\partial^2 z}{\partial x^2} = 4\frac{\partial^2 z}{\partial y^2}$ and $z = \frac{\partial z}{\partial y} = sinx$ when $y = 0$ 5

(c) Find: $Z[3^n]$, $n \geq 1$ 5

9. (a) Find: $Z^{-1}\left[\frac{z+1}{z^2-2z+1}\right]$ 5

(b) Apply Runge-kutta fourth order method to find approximate value of y when

$x = 0.1$, given that $\frac{dy}{dx} = x + y^2$ and $y = 1$ when $x = 0$, take h = 0.1 5

(c) Solve by z-transform: $f(n) + \frac{1}{4} f(n-1) = u(n) + \frac{1}{3} u(n-1)$ 5

10. (a) Use Lagrange's formula to find polynomial $f(x)$ for the following data: 5

x	$f(x)$
0	4
1	3
4	24
5	39

(b) Find: $Z\left[\left(\frac{1}{4}\right)^{|n|}\right]$ for all n 5

(c) Solve: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ 5

the conditions are :

(i) $T(0, y) = T_0$ for $0 < y < 1$

(ii) $T(x, 0) = T(x, l) = 0$

(iii) $T(\infty, y) = 0$

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(Revised Course)

FACULTY OF ENGONEERING AND TECHNOLOGY

S.Y.B.Tech.(All) EXAMINATION

MAY/JUNE, 2014

ENGINEERING MATHEMATICS-IV

Time-Three Hours Maximum Marks-80

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(ii) From section A solve any two section questions from the remaining Q.No.2,3,4 and 5

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(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks) 10

(a) Cauchy-Riemann equation in polar coordinates are …………

(b) Evaluate: $\int_0^{2i} coshz\, dz$

(c) Evaluate: $\int_C \frac{Logz}{z-3} dz\,,\ C:\ |z| = 4$

(d) If $f(z)$ has a pole of order Four at $z = a$ then $Resf(a) =$ ……….

(e) Define bilinear transformation.

(f) A necessary and sufficient condition that the integral $\int_c \overline{F}\, d\overline{r}$ vanishes is that ………

(g) State Stoke's theorem

(h) If $\overline{F} = (2xy + z^3)i + x^2 j + 3z^2 k$ then check whether $\overline{F}$ is conservative.

2. (a) Prove that an analytic function with constant amplitude is constant. 5

(b) Find the work done when a force $\overline{F} = x^2 i + xyj$ moves a particle in x-y plane from $(0,0)$ to $(1,1)$ along the path $y^2 = x$ 5

(c) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$ 5

3. (a) Show that $u = x^2 - y^2 - y$ is harmonic function. Find the conjugate harmonic function and corresponding analytic function. 5

(b) Apply Green's theorem to evaluate $\int_C [e^{-x} sinydx + e^{-x} cosydy]$ where C is the rectangle whose vertices are $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$ 5

(c) Evaluate: $\int_0^{3+i} z^2 dz$ along the curve $x = 3y$ 5

4. (a) Evaluate: $\int_C \frac{e^{-z}}{(z+2)^5} dz$ where C is the circle $|z| = 3$ by Cauchy's integral formula 5

(b) Apply Stokes-theorem evaluate $\int_C [(x + 2y)dx + (x - z)dy + (y - z)dz]$ where C is the boundary of the triangle with vertices (2, 0, 0) , (0 , 3, 0) and (0, 0, 6). 5

(c) Expand $\frac{1}{(z+1)(z+3)}$ for $1 < |z| < 3$ in Laurent series 5

Q5 (a) Evaluate: $\int_C \frac{(1-2z)}{z(z-1)(z-2)} dz$, C : $|z| = 1.5$ by Cauchy's Residue theorem. 5

(b) Evaluate: $\iint_S \overline{F}\, \hat{n}\, ds$ as a volume integral if $\overline{F} = x^3 i + y^3 j + z^3 k$ over $x^2 + y^2 + z^2 = a^2$ 5

(c) Find the image of the circle $|z - 1| = 1$ under the mapping $w = \frac{1}{z}$

Section B

6. Solve any five (Each for two marks) 10

(a) Find $Z[u(n)]$

(b) Find: $Z[sin2n\ cos4n]$

(c) Find $Z^{-1}\left[\frac{z}{(z-1)^2}\right]$

(d) Find $f(3)$ for the following data:

x	$f(x)$
0	4
1	3
4	24

(e) Equation $3x = cosx + 1$ is required to be solved by Newton-Raphson method with initial guess $x_0 = 1$. Then find first approximation of the root (x_1) correct to 3 decimal places.

(f) Find the values of k_1 and k_2 while solving the differential equation $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$ by Runge-Kutta fourth order method.

(g) If $u = C\ e^{kx+(3-4k)y}$ is the solution of $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, $u(0, y) = 3e^{-y}$ then find the value of c and k

(h) The three possible solution of Laplace equation $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ are ……….

7. (a) Solve: $4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u$ given $u(x, 0) = 3e^{-x} - e^{-5x}$

(b) Solve by Gauss-Seidel method

$$28x + 4y - z = 32$$

$$x + 3y + 10z = 24$$

$$2x + 17y + 4z = 35$$

(c) Find $y(0.1)$ by Taylor's series method, given $\frac{dy}{dx} = x - y^2, y(0) = 1$

(Find the value correct to 4 decimal places)

8. (a) Use Newton-Raphson method to find a root of the equation $Log_e x - x + 3 = 0$

(Calculate correct up to 3 decimal places)

(b) If $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends ,find the solution with conditions $y(0,t) = y(L,t) = 0, y(x,0) = y_0 \, sin\frac{\pi x}{L}$ and $\frac{\partial y}{\partial t} = 0$ at $t = 0$

(c) Find: $Z\left[\sin\left(\frac{n\pi}{2} + \frac{\pi}{3}\right)\right]$

9. (a) Find: $Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}\right]$

(b) Use Runge-kutta fourth order method to find $y(0.1)$ given that:

$\frac{dy}{dx} = 3e^x + 2y, y(0) = 0$, and $h = 0.1$

(c) Solve: $f(n+2) = 2[f(n+1) + f(n)]$, $f(0) = 0$, $f(1) = 1$ by z-transform

10. (a) In the steady state temperature u satisfies the differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

the conditions are:

(i) $u = 0$ when $x = 0$

(ii) $u = 0$ when $x = \pi$

(iii) $u = 0$ when $y \to \infty$

(iv) $u = u_0$ when $y = 0$ for $0 < x < \pi$

(b) Use Lagrange's interpolation formula to find $f(0.6)$ for given data

x	$f(x)$
0.4	-0.916
0.5	-0.693
0.7	-0.357
0.8	-0.223

(c) Find: $Z\left[\frac{\alpha^n - \beta^n}{n}\right]$, $n \geq 0$

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(Revised Course)

FACULTY OF ENGONEERING AND TECHNOLOGY

S.Y.B.Tech.(All) EXAMINATION

NOVEMBER/DECEMBER, 2014

ENGINEERING MATHEMATICS-IV

Time-Three Hours Maximum Marks-80

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(iii) From section B solve any two section questions from the remaining Q.No.**7,8,9** and **10**

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(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks) 10

(a) If $f(z)$ is analytic within and on a closed curve C and if 'a' is any point within C , then:

$$\int_C \frac{f(z)}{(z-a)^{n+1}}\, dz = \ldots\ldots\ldots\ldots..$$

(b) Determine b such that: $u = e^{bx} cos7y$ is harmonic.

(c) Find the poles of $cotz$

(d) Evaluate: $\int_C \frac{dz}{2z-3}$, $C : |z| = 1$

(e) If $z = cos\theta + i\, sin\theta$, $\frac{1}{z} = cos\theta - isin\theta$ then $sin\theta = \ldots\ldots\ldots$ and $cos\theta = \ldots\ldots\ldots$

(f) State Green's theorem

(g) Find the value of $\int grad(x + y - z)dR$, from $(0,1,-1)$ to $(1,2,0)$

(h) $\int_c \overline{F}\, d\overline{r}$ is independent of the path joining any two points if and only if it is ……….

2. (a) If $f(z) = u + iv$ is an analytic function and $u - v = e^x(cosy - siny)$ find $f(z)$ in terms of z. 5

(b) Evaluate: $\int_{-2}^{-2+i}(2+z)^2 dz$ along the straight line joining $z = -2$ to $z = -2 + i$ 5

(c) Find the work done in moving a particle once round an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ in the plane $z = 0$ in the force field given by: $\overline{F} = (3x - 2y)i + (2x + 3y)j + y^2 k$ 5

3. (a) Show that $f(z) = \left(r + \frac{1}{r}\right) cos\theta + i\left(r - \frac{1}{r}\right) sin\theta$ is analytic.Also find $f(z)$ in terms of z 5

(b) Use Cauchy's integral formula to evaluate: $\int_C \frac{z}{z^2-3z+2} dz$, C : $|z-2| = \frac{1}{2}$ 5

(c) Apply Stoke's theorem to evaluate $\int (x^2 + y - 4)dx + 3xydy + (2xz + z^2)dz$ over the surface of the hemisphere $x^2 + y^2 + z^2 = 9$ 5

4. (a) Evaluate: $\int_C \frac{e^z}{cos\pi z} dz$, C : $|z| = 1$ 5

(b) Find the bilinear transformation which maps the points $z = \infty, i, 0$ on to $w = 0, i, \infty$ 5

(c) Apply Green's theorem to evaluate: $\int_C \overline{F}\, d\,\overline{r}$ where $\overline{F} = x^2 i + xyj$ and C is a triangle having vertices: A (0, 2), B (2, 0) and C (4, 2) 5

5. (a) Expand $f(z) = \frac{1}{z(z+2)^3}$ by Laurent series about $z = -2$ 5

(b) Find the image of the circle $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$ 5

(c) Evaluate: $\iint_s \overline{F}\,\hat{n}\, ds$ where $\overline{F} = 4xi - 2y^2 j + z^2 k$ and S is the region bounded by $y^2 = 4x\,, x = 1\,, z = 0\,, z = 3$ 5

Section B

6. Solve any five (Each for two marks) 10

(a) Find $Z\left[\frac{a^n}{n!}\right]$, $n \geq 0$

(b) Find $z[n]\,, n \geq 0$

(c) Find $Z^{-1}\left[\frac{z}{z-a}\right]$

(d) Write formula to solve: $\frac{dy}{dx} = f(x,y)\,, y(x_0) = y_0$ by Runge-Kutta IVth order method.

(e) Find the values of x, y, z in the first iteration for the system of equations

$9x + 3y + 4z = 9$

$2x + 8y - 3z = 9$

$2x - 3y + 10z = 26$

to be solved by Gauss-Seidel method

(f) The real root of the equation $x\,e^x = 2$ is evaluated by Newton's Raphson's method.If the initial guess $x_0 = 0.867$, then find first approximation of the root (x_1)

(g) If $u = Ce^{kx+\frac{1}{2}(k-1)t}$ is the solution of: $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial t} = u\,,\; u(x,0) = 6e^{-3x}$

(h) The general solution of one-dimensional heat flow equation when both ends of the bar are kept at zero temperature is of the form …………..

7. (a) Solve: $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} cosx$ 5

(b) Solve by Gauss elimination method

$2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16$ 5

(c) Given $\frac{dy}{dx} = 3x + y^2\,, y(0) = 1$ find $y(0.2)$ by Taylor;s series method. 5

8. (a) Find by Newton-Raphson's method the root of the equation: $Logx - cosx = 0$

(Correct to three decimal places) 5

(b) A string is stretched tightly between $x = 0$ and $x = l$ and both the ends are given the 5 displacement $y = k\ sinpt$ perpendicular to the string satisfies $\frac{\partial^2 y}{\partial t^2} = a^2\ \frac{\partial^2 y}{\partial x^2}$ then find y

(c) Find: $Z\left[\left(\frac{1}{5}\right)^n\right]$ $, n < 0$

9. (a) Find $Z^{-1}\left[\frac{z^3}{\left(z-\frac{1}{2}\right)(z-1)^2}\right]$ 5

(b) Use Runge-Kutta method of Fourth order to find $y(0.8)$ with $h = 0.4$ for :

$\frac{dy}{dx} = \sqrt{x+y}\, , y(0.4) = 0.41$ 5

(c) Solve by z-transform

$f(n) - 2f(n-1) + f(n-2) = 1\ ,\ \ n \geq 0$

$= 0\ , n < 0$ 5

10. (a) A rod of length L with insulated sides is initially at uniform temperature u_0, its ends are suddenly cooled to 0^0 C and are kept at that temperature. Find the temperature function $u(x, t)$. 5

(b) Find $Z[2^n(n^2 + 3n + 2)]$ 5

(c) Use Lagrange's formula to find $f(5)$ for: 5

x	$f(x)$
0	648
2	704
3	729
6	792

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(Revised Course)

FACULTY OF ENGONEERING AND TECHNOLOGY

S.Y.B.Tech.(All) EXAMINATION

MAY/JUNE, 2015

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(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks) 10

a) Cauchy-Riemann equation in Cartesian form are …………….

b) If $f(z) = e^z$ then the expansion of $f(z)$ about $z = a$ in Taylor's series is …………

c) Evaluate: $\int_C \frac{dz}{z^2 C^z}$ where C:$|z| = 1$

d) Show that $u = x^2 - y^2 - y$ is harmonic.

e) Find the image of infinite strip $x > 0$ under the transformation $w = \frac{1}{z}$ and represent graphically.

f) State Green's theorem

g) Show that $\overline{F} = (y^2 cosx + z^3)i + (2y\, sinx - 4)j + (3xz^2 + 2)k$ is conservative.

h) Find the value of $\int grad(x + y - z)dr$, from $(0,1,-1)$ to $(1,2,0)$

2. (a) Determine the analytic function whose real part is $y + e^x cosy$. 5

(b) If a force $\overline{F} = 2x^2yi + 3xyj$ displaces a particle in the xy-plane from (0, 0) to (1, 4) along a curve $y = 4x^2$ find the work done. 5

(c) Find the bilinear transformation that maps the point $z = 0,1, i$ in z-plane on to the point $w = 1 + i, -i , 2 - i$ in the w-plane. 5

3. (a) In two dimensional heat flow stream function $\Psi = -\frac{y}{x^2+y^2}$ then find the velocity ϕ 5

(b) If $\overline{F} = x^3i + y^3j + z^3k$ and S is the surface of sphere $x^2 + y^2 + z^2 = a^2$. Verify Gauss divergence theorem 5

(c) Evaluate: $\int_C \frac{dz}{z^3(z+4)}$ where C is the circle $|z| = 2$ 5

4. (a) Evaluate: $\int_C \frac{15z+9}{z(z^2-9)}\, dz$ where C is the circle $|z - 1| = 3$ 5

(b) Evaluate $\iint_S (x^3dydz + x^2ydzdx + x^2zdxdy)$ where S surface bounded by the cylinder $z = 0 , z = 3$ & $x^2 + y^2 = 4$ 5

(c) If $f(z) = -x + y - 3x^2i$ & C is the straight line joining origin to (1, 1) then evaluate $\oint f(z)dz$ 5

5 (a) Prove that: $v = r^n sinn\theta$ is harmonic. Find conjugate harmonic function u and hence corresponding analytic function. 5

(b) Evaluate $\iint_S \overline{F}\,\hat{n}\ ds$ where $\overline{F} = yi + 2xj - zk$ and S is the surface of the plane $2x + y = 6$ in the first octant cut at by the plane $z = 4$ 5

(c) Using calculus of residue find $\int_0^{2\pi} \frac{cos2\theta}{5+4cos\theta} d\theta$ 5

Section B

6. Solve any five (Each for two marks) 10

(a) Find $Z\{3^n \cos(2-5n)\}, n \geq 0$

(b) For $n \geq 0, Z(n^2) =$……

(c) For $n \geq 0$, $Z(e^{-an}3^n) =$…….

(d) Newton-Raphson method is $x_{n+1} =$ ………..

(e) Langrage's interpolation formula for unequal interval is $y =$………

(f) Using Picard's method first approximation for the equation $\frac{dy}{dx} = 1 + xy, y(0) = 1$

$y_1 =$………..

(g) The solution of 3 $\frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$ is $u =$ ……….

(h) The solution of $\frac{\partial^2 u}{\partial x \partial y} = sinx\ siny$ is $u =$……..

7.(a) Solve the partial differential equation $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ of the vibrating string which is stretched and fastened at two point l apart .Under the condition

(i) $u(0,t) = 0$ (ii) $u(l,t) = 0$ (iii) $\frac{\partial u}{\partial t} = 0$, $t = 0$ (iv) $u(x,0) = k(lx - x^2)$ 5

(b) Using Newton-Raphson method, find the real root of $xe^x - 2 = 0$ 5

(c) Find the value of y for $x = 0.1$ by Picard's method given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ 5

8. (a) Find $y(1.2)$ by Runge-Kutta method of fourth order of $\frac{dy}{dx} = \sqrt{x^2 + y^2}$ with

$y(1) = 1.5$ take $h = 0.1$ 5

(b) Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ if (i) $u(0,t) = 0$ (ii) $u(l,t) = 0$ (iii) $u(x,t)$ is bounded

(iii) $u(x,0) = \frac{u_0 x}{l}$, $for\ 0 \leq x \leq l$ 5

(c) Find $Z\left[\frac{sinan}{n}\right]$, $n \geq 0$ 5

9. (a) Solve: $f(n+2) + 6\,f(n+1) + 9\,f(n) = 2^n$ with $f(0) = 0$, $f(1) = 0$ using z-Transform 5

(b) Apply Runge-kutta fourth order method to find approximate value of y when $x = 0.2$, given that $\frac{dy}{dx} = x + y^2$, $y = 1$ when $x = 0$, take h = 0.1 5

(c) Find $Z^{-1}\left[\frac{z^3}{\left(z-\frac{1}{2}\right)^2 (z-1)}\right]$ 5

10. (a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the condition i) $u(0, y) = 0$ (ii) $u(\pi, y) = 0$ iii) $u(x, \infty) = 0$ iv) $u(x, 0) = u_0$ for $0 < x < \pi$ 5

(b) Solve by Gauss-Seidal method $6x + y + z = 105$, $4x + 8y + 3z = 155$ 5

$5x + 4y - 10z = 65$

(c) Find $Z[f(n)]$ if $f(n) = \begin{cases} 9^n, n < 0 \\ 5^n, n \geq 0 \end{cases}$ also find region of convergence. 5

CODE NO. : K-1108-2015

(Revised Course)

FACULTY OF ENGONEERING AND TECHNOLOGY

S.Y.B.Tech. (All) EXAMINATION

NOVEMBER/DECEMBER, 2015

ENGINEERING MATHEMATICS-IV

Time-Three Hours Maximum Marks-80

''Please check whether you have got the right question paper.''

N.B. :- (i) Question No.1 and Question No.6 are compulsory.

(ii) From section A solve any two section questions from the remaining Q.No.2,3,4 and 5

(iii) From section B solve any two section questions from the remaining Q.No.7,8,9 and 10

(iv) Figures to the right indicate full marks.

(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks) 10

(a) If $w = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$ is analytic then $a =$...... and $b =$ ....

(b) Harmonic function in polar form is ....

(c) If $f(z) = cosz$ then the expansion of $f(z)$ about $z = 0$ in Taylor's series is ....

(d) Evaluate: $\oint \frac{e^{2z}}{(z+1)^4}\, dz$ where C: $|z| = 2$

(e) Find the image of $y < 1$ under the transformation $w = \frac{1}{z}$

(f) Define conservative field.

(g) State stokes theorem.

(h) If $\overline{F}$ represents the variable force acting on a particle along arc AB then the total work done is

2. (a) Determine whether $w = logz$ is analytic or not, if analytic find $\frac{dw}{dz}$ 5

(b) Find the work done in moving a particle one round the circle $x^2 + y^2 = a^2$under the field of force $\overline{F} = (siny)i + (1 + cosy)j$ 5

(c) Find the bilinear transformation which maps the points $z = 0, -i, 2i$ in z-plane onto the points $w = 5i, \infty, -\frac{i}{3}$ in w-plane. 5

3. (a) In two dimensional fluid flow the stream function $\Psi = tan^{-1}\left(\frac{y}{x}\right)$ is given find the velocity ϕ 5

(b) Evaluate $\iint_s \overline{F}\ ds$ where S is closed surface bounded by the planes $z = 1$, $z = 1$ and cylinder $x^2 + y^2 = 1$ where $\overline{F} = xi - yj + (z^3 - z)k$ 5

(c) Evaluate $\int_0^{3+i} z^2\ dz$ along the curve $x = 3y$ 5

4. (a) If C: $|z - 2| = 2$ and $f(z) = \frac{z^2+4}{(z-2)(z+3i)}$ then evaluate $\oint f(z)dz$ 5

(b) Using Stokes theorem evaluate: $\int_C [(2x - y)dx - yz^2dy - y^2zdz]$ where C is the circle $x^2 + y^2 = 1$ corresponding to the surface of unit radius. 5

(c) If $f(z) = x - 2iy$ and C: boundary of the square with vertices $(1,1), (2,1), (2,2)$ and $(1,2)$ taken anticlockwise sense then find integration of $f(z)$ 5

5 (a) Prove that $V = r^n sinn\theta$ is harmonic. Find conjugate harmonic function u and hence corresponding analytic function. 5

(b) Evaluate $\iint_s \overline{F}\,\hat{n}\ \ ds$ where $\overline{F} = xi + yj + zk$ and S is the triangle $(1,0,0), (0,1,0), (0,0,1)$ 5

(c) Evaluate: $\int_0^{2\pi} \frac{d\theta}{(5-3sin\theta)^2}$ by using residue. 5

Section B

6. Solve any five (Each for two marks) 10

(a) Solve $z\{sin2n - sin7n\}, n \geq 0$

(b) If $z\{f(n)\} = \overline{f}(z)$ then $z\{e^{an}f(n)\} =$ ……….

(c) For $n \geq 0$, $z\{3^n e^{3n}\}$ is …….

(d) Using Picard's method first approximation for the equation $\frac{dy}{dx} = x + y, y(0) = 1$ is

$y_1 =$……….

(e) The order of convergence in Newtom-Raphson method is ………

(f) Find $f(2)$ for the following data:

x	$f(x)$
0	4
1	3
4	24

(g) The one dimensional heat equation is represent by partial differential equation ……….

(h) The solution of $\frac{\partial^2 u}{\partial x\, \partial t} = e^{-t} cosx$ is $u =$……..

7. (a) Solve: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ if : $y(0,t) = 0$, $y(l,t) = 0$, $y(x,0) = f(x)$ and

$\frac{\partial}{\partial t} y(x,0) = 0,\ 0 < x < 1$ 5

(b) Find the real root of $x^3 - 4x - 9 = 0$ by using Newton-Raphson method upto five decimal places. 5

(c) Find $y(1.2)$ by using Runge-Kutta method of fourth order of $\frac{dy}{dx} = \sqrt{x^2 + y^2}$ with

$y(1) = 1.5$ take $h = 0.1$ 5

8. (a) Find by Picard's method $y(0.2)$ given that: $\frac{dy}{dx} = 1 + y^2$ and $y(0) = 1$ 5

(b) Solve: $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 5u$ when $u(x, 0) = 5e^{-3x} + 7e^{2x}$

(c) Solve: $y_{n+1} + \frac{1}{4}y_n = \left(\frac{1}{4}\right)^n$, $n \geq 0$ with $y_0 = 0$ 5

9. (a) Find: $z\{3n^2 - 4n + 5\}, n \geq 0$ 5

(b) Apply Euler's modified method to find the approximate value of $y(0.3)$. Given that

$\frac{dy}{dx} = x + y$ and $y(0) = 1$ 5

(c) Find $Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}\right]$; $n \geq 0$ 5

10. (a) Find : $z\left[\frac{a^n - b^n}{n}\right], n \geq 0$ 5

(b) Solve: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the condition

(i) $u = 0$ when $y \to \infty$

(ii) $u = 0$ when $x = 0$ for all y

(iii) $u = 0$ when $x = 1$

(iv) $u = x(1 - x)$ when $y = 0$ for $0 < x < 1$ 5

(c) Solve by Gauss-Seidal method: 5

$11x - y + 2z = 40$

$x - 3y + 13z = 42$

$2x + 15y - z = 30$

Compiled & Collected by
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