CODE NO. : P-1148-2013

(Revised Course)

FACULTY OF ENGINEERING AND TECHNOLOGY

F.Y.B.Tech.(All) EXAMINATION

NOVEMBER/DECEMBER, 2013

ENGINEERING MATHEMATICS-I

Time-Three Hours Maximum Marks-80

''Please check whether you have got the right question paper.''

N.B. :- (i) Question No.1 and Question No.6 are compulsory.

(ii) From section A solve any two section questions from the remaining Q.No.2, 3, 4 and 5

(iii) From section B solve any two section questions from the remaining Q.No.7, 8, 9 and 10

(iv) Figures to the right indicate full marks.

(v) Assume suitable data if necessary.

Section A

1. Solve any five (each for two marks) 10

(a) Discuss the conditions for consistency of given system of non-homogeneous linear eqations.

(b) Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ -3 & 4 & 5 \\ 7 & -1 & 6 \end{bmatrix}$

(c) Define linear dependence and linear independence of the vectors.

(d) Evaluate: $\lim_{x \to 0} x\ Logx$

(e) Prove that: $f(mx) = f(x) + (m-1)x\, f'(x) + \frac{(m-1)^2}{2!} x^2 f''(x) + \frac{(m-1)^3}{3!} x^3 f'''(x) + \cdots$

(f) If $y = e^x sin3x$ then find y_n

(g) If $u_n = \sum_{n=1}^{\infty} (-1)^n$ then $\lim_{n \to \infty} u_n =$ …………

(h) The comparison test states that ……………

2. (a) Reduce the following matrix o its normal form :

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$ 5

(b) Find the nth derivative of: $y = e^{2x} sin4x \ cos2x$ 5

(c) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{2^n}{n^3+1}$ 5

3. (a) Find Eigen values and Eigen vector corresponding to the smallest eigen value for matrix:

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$ 5

(b) Evaluate: $\lim_{x \to \infty} [cosh^{-1}x - Logx]$ 5

(c) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ 5

4. (a) Test the consistency and hence solve:

$$x + y + z = 6$$

$$x - y + 2z = 5$$

$$3x + y + z = 8$$

$$2x - 2y + 3z = 7$$ 5

(b) Discuss the convergence of the series: $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$ 5

(c) Prove that: $e^{\theta} = 1 + sin\theta + \frac{1}{2!} \ sin^2\theta + \frac{2}{3!} \ sin^3\theta + \cdots$ 5

5. (a) Verify Cayley-Hamilton theorem for :

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$ 5

(b) If $y = \left[x - \sqrt{x^2 - 1}\,\right]^m$ then prove that:

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0$$ 5

(c) Test the convergence of the series: $\frac{1}{3^p} + \frac{1}{5^p} + \frac{1}{7^p} + \cdots$ 5

Section B

6. Solve any five (each for two marks) 10

(a) Find the locus of: $|z + 1 - i| = 3$

(b) Separate into real and imaginary part of: $\sin(2x + iy)$

(c) Prove that: $log i = i\left(2\pi + \frac{\pi}{2}\right)$

(d) If $y^x + x^x = (x + y)^{(x+y)}$ find $\frac{dy}{dx}$

(e) If $u = lx + my, v = mx - ly$ find $\left(\frac{\partial x}{\partial u}\right)_v$

(f) If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then find: $x\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y}$

(g) If $x = e^{\theta} cos\phi\,, y = e^{\theta} sin\phi$ then find $\frac{\partial(x,y)}{\partial(\theta,\phi)}$

(h) State the sufficient condition for existence of maxima and minima.

7. (a) $(1 + cos\theta + i\, sin\theta)^n + (1 + cos\theta - i\, sin\theta)^n = 2^{n+1} cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$ 5

(b) If $x = \frac{cos\theta}{u}\,, y = -\frac{sin\theta}{u}$ and $z = f(x, y)$ show that:

$$\left(\frac{\partial x}{\partial u}\right)_\theta \left(\frac{\partial u}{\partial x}\right)_y = cos^2\theta$$ 5

(c) If $x = u(1 - v), y = uv$ then prove that: $J\,J' = 1$ 5

8. (a) If $log_e log_e(x + iy) = f + iq$ then prove that :

$$y = x\tan\left\{(tanq)\, log_e\sqrt{x^2 + y^2}\right\}$$ 5

(b) If $u = \frac{e^{x+y+z}}{e^x+e^y+e^z}$ find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ 5

(c) Prove that: $sin7\theta = 7sin\theta - 56\, sin^{3}\theta + 112\, sin^{5}\theta - 64\, sin^{7}\theta$ 5

9. (a) If $x = 2cos\theta\, cosh\phi$, $y = 2sin\theta\, sinh\phi$ prove that:

$sec(\theta + i\phi) + sec(\theta - i\phi) = \frac{4x}{x^2+y^2}$ 5

(b) If $u = sin^{-1}\left(\frac{x^{1/4}-y^{1/4}}{x^{1/5}+y^{1/5}}\right)$

prove that: $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\,\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{tanu}{400}\,[tan^2u - 19]$ 5

(c) Apply De Moivre's theorem to solve:

$x^7 + x^4 + x^3 + 1 = 0$ 5

10. (a) Prove that: $tan^{-1}z = \frac{i}{2}\,log\frac{i+z}{i-z}$ 5

(b) If $z = f(x, y)$ where $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ then prove that:

$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\,\frac{\partial z}{\partial x} - y\,\frac{\partial z}{\partial y}$ 5

(c) Discuss the maxima and minima of: $x^3y^2(1 - x - y)$ 5

CODE NO. : K-1001-2014

(Revised Course)

FACULTY OF ENGINEERING AND TECHNOLOGY

F.Y.B.Tech.(All) EXAMINATION

MAY/JUNE, 2014

ENGINEERING MATHEMATICS-I

Time-Three Hours Maximum Marks-80

‘‘Please check whether you have got the right question paper.’’

N.B. :- (i) Question No.1 and Question No.6 are compulsory.

(ii) From section A solve any two section questions from the remaining Q.No.2,3,4 and 5

(iii) From section B solve any two section questions from the remaining Q.No.7,8,9 and 10

(iv) Figures to the right indicate full marks.

(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks) 10

(a) Define Rank of matrix.

(b) Find eigen values of: $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

(c) For system of linear homogeneous equations:

(i) $|A| = 0$, then there exists ……………..

(ii) $|A| \neq 0$, then there exists ……………..

(d) If $y = sin^{2}x$ then find y_n

(e) Evaluate: $\lim_{x \to 0} x\, logx$

(f) If $y = sin^{-1}x$ then prove that: $(1 - x^2)y_2 - xy_1 = 0$

(g) Test the convergence of the series: $x + 2x^2 + 3x^3 + \cdots$

(h) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

2. (a) Test for consistency and solve:

$$x + 2y + z = 3$$

$$2x + 3y + 2z = 5$$

$$3x - 5y + 5z = 2$$

$$3x + 9y - z = 4$$ 5

(b) Prove that: $e^{xcos\alpha} \cos(xsin\alpha) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \cos(n\alpha)$ 5

(c) Test for convergence of series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots \infty$ 5

3. (a) Find Eigen values and Eigen vector corresponding to the largest eigen value if:

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$ 5

(b) State the values of x for which the series converges :

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots\infty$$ 5

(c) Evaluate: $\lim_{x \to 1} (1 - x^2)^{\frac{1}{Log(1-x)}}$ 5

4. (a) Find A^{-1} by Cayley-Hamilton theorem if:

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$ 5

(b) Find Taylor's series expansion for $Logcosx$ about the point $\frac{\pi}{3}$ 5

(c) Test for convergence of the series: $\sum \frac{(n!)^2}{(2n)!} x^{2n}$ 5

5. (a) Are the following vectors linearly dependent ? If so, find the relation between them:

$x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$

$x_3 = (0, 1, 2)$, $x_4 = (-3, 7, 2)$ 5

(b) If $y = \sin[Log(x^2 + 2x + 1)]$ Prove that

(i) $(x+1)^2 y_2 + (x+1)y_1 + 4y = 0$
(ii) $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$ 5

(c) Test the convergence of the series: $\sum \frac{(n+1)(n+2)}{(n^2+1)(n^2+2)}$ 5

Section B

6. Solve any five (Each for two marks) 10

(a) Find the locus of : $|z - 2i| = 3$

(b) If $z = 1 + i$, then find $|z|$ and $\mathrm{amp}(z)$

(c) Find the general value of $Log(-5)$

(d) If $u = \frac{e^{x+y+z}}{e^x + e^y + e^z}$ find $\frac{\partial u}{\partial y}$

(e) If $u = \frac{x^2 y^2 z^2}{x^2 + y^2 + z^2}$ then apply Euler's theorem to find: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

(f) If $z = f(x, y)$ and $x = u + v, y = 4v$ then find $u\frac{\partial z}{\partial u}$

(g) If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

(h) State under what condition there exists maxima for: $u = f(x, y)$

7. (a) If α and β are the roots of equation $x^2 - \sqrt{3}\,x + 1 = 0$ prove that

$\alpha^n + \beta^n = 2\cos\left(\frac{n\pi}{6}\right)$ 5

(b) If $z = e^{ax+by} f(ax - by)$ prove that: $a\frac{\partial z}{\partial y} + b\frac{\partial z}{\partial x} = 2abz$ 5

(c) If $x = e^v\, secu$, $y = e^v\, tanu$ then prove that: $J\,J' = 1$ 5

8. (a) Separate into real and imaginary parts of: $\frac{cosz}{z+1}$ 5

(b) If $x^x y^y z^z = c$ show that at $x = y = z$, $\frac{\partial^2 z}{\partial x\, \partial y} = -\left[\frac{1}{x(1+Logx)}\right]$ 5

(c) Examine the function:

$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values 5

9. (a) Prove that: $Log\left(\frac{a+ib}{a-ib}\right) = 2i\ tan^{-1}\left(\frac{b}{a}\right)$ 5

(b) If $u = tan^{-1}\left(\frac{\sqrt{x^3+y^3}}{\sqrt{x}+\sqrt{y}}\right)$ find the value of:

$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\ \frac{\partial^2 u}{\partial x\, \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ 5

(c) If $x = 2cos\theta\ cosh\phi$, $y = 2sin\theta\ sinh\phi$ prove that:

$sec(\theta + i\phi) - i\ sec(\theta - i\phi) = \frac{4iy}{x^2+y^2}$ 5

10. (a) Prove that: $tan^{-1} i\left(\frac{x-a}{x+a}\right) = \frac{i}{2}\ Log\left(\frac{x}{a}\right)$ 5

(b) If $z = f(x, y)$, $x + y = 2e^{\theta}\ cos\phi$, $x - y = 2i\ e^{\theta}\ sin\phi$ then show that:

$\frac{\partial^2 z}{\partial \theta^2} + \frac{\partial^2 z}{\partial \phi^2} = 4xy \frac{\partial^2 z}{\partial x\, \partial y}$ 5

(c) Examine: $f(x, y) = x^3 + y^3 - 3axy$ for maxima and minima. 5

CODE NO. : U-1272-2014

(Revised Course)

FACULTY OF ENGINEERING AND TECHNOLOGY

F.Y.B.Tech.(All) EXAMINATION

NOVEMBER/DECEMBER, 2014

ENGINEERING MATHEMATICS-I

Time-Three Hours Maximum Marks-80

‘‘Please check whether you have got the right question paper.’’

N.B. :- (i) Question No.1 and Question No.6 are compulsory.

(ii) From section A solve any two section questions from the remaining Q.No.2,3,4 and 5

(iii) From section B solve any two section questions from the remaining Q.No.7,8,9 and 10

(iv) Figures to the right indicate full marks.

(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks) 10

(a) Define Characteristic equation of matrix.

(b) Rank of matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is ………………..

(c) Define linear dependence and independence of vectors.

(d) Rabbe’s test sates that ……………

(e) If $y = sin3x\ cos4x$ then $y_n =$ ………………

(f) Examine convergence of the series: $\sum_{n=1}^{\infty} \frac{1}{2^n}$

(g) Using Maclaurin’s series expand $sinx$

(h) Evaluate: $\lim_{x\to 0} x\, logx$

2. (a) Reduce the following matrix to its normal form and hence find rank of matrix :

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & 1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$ 5

(b) Test the convergence of the series: $\sum \frac{(n+1)(n+2)}{(n^2+1)(n^2+2)}$ 5

(c) Prove that: $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} \dots\right)$ 5

3. (a) Find for what values of k, the following system of equations

$$2x - 3y + 6z - 5t = 3$$

$$y - 4z + t = 1$$

$$4x - 5y + 8z - 9t = k$$

has : (i) No solution

(ii) Infinite no. of solution 5

(b) If $y = \frac{x}{x^2+a^2}$ find y_n 5

(c) Discuss convergence of series: $\sum_{n=1}^{\infty} \frac{(n+1)^{n^2}}{n^{n^2}\, 3^n}$ 5

4. (a) Find eigen values and eigen vectors corresponding to the smallest eigen value of matrix

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$ 5

(b) Evaluate: $\lim_{x\to 0} \frac{e^{2x}-(1+x)^2}{x\log(1+x)}$ 5

(c) Test convergence of series: $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \cdots$ 5

5. (a) Examine for linear dependence, independence of the following vectors. If dependent find relation between them

$x_1 = [1, 2, -1, 0]$

$x_2 = [1, 3, 1, 2]$

$x_3 = [4, 2, 1, 0]$

$x_4 = [6, 1, 0, 1]$ 5

(b) Expand: $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$ 5

(c) Test convergence of the series whose nth term is: $u_n = \frac{1}{n^{\frac{a+b}{n}}}$ 5

Section B

6. Solve any five (Each for two marks) 10

(a) Find locus of z satisfying: $|z - i| = 6$

(b) Find general value of $\log(-\sqrt{3})$

(c) Find modulus and amplitude of $1 - i\sqrt{3}$

(d) Find $\frac{dy}{dx}$ if $x^y + y^x = C$

(e) If $u = \frac{x+y+z}{\sqrt{x}+\sqrt{y}+\sqrt{z}}$ then find: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

(f) If z is function of u and v and $u = lx + my$, $v = ly - mx$ then find values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

(g) If $u = 3x + 5y$, $v = 4x - 3y$ then value of $\frac{\partial(u,v)}{\partial(x,y)}$ is …………

(h) If u, v are functions of r, s and r, s are functions of x, y then $\frac{\partial(u,v)}{\partial(x,y)} =$ …………

7. (a) Prove that: $sech^{-1}(sin\theta) = logcot\frac{\theta}{2}$ 5

(b) If $v = \frac{c}{\sqrt{t}} e^{-\frac{x^2}{4a^2t}}$ then show that: $\frac{\partial v}{\partial t} = a^2\frac{\partial^2 v}{\partial x^2}$ 5

(c) Find extreme values of : $u = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$ 5

8. (a) Separate $i^{(1+i)}$ into real and imaginary parts. 5

(b) If $x = u\ tanv\ , y = u\ secv$ prove that: $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x$ 5

(c) Verify $JJ' = 1$ for function: $x = u\ , y = u\ tanv\ , z = w$ 5

9. (a) If $\tan(x + iy) = i$ where x and y are real, prove that x is indeterminate and y is infinite. 5

(b) Prove that: $\frac{sin6\theta}{sin\theta} = 32\ cos\ ^5\theta$ 5

(c) If $u = tan\ ^{-1}\left(\frac{y}{x}\right)\ , x = e^t - e^{-t}\ , y = e^t + e^{-t}$ find $\frac{du}{dt}$ 5

10. (a) Find all values of $(1 + i)^{1/5}$. Show that their product is $1 + i$ 5

(b) Prove that: $cosh\ ^{-1}x = \pm\ \log\left(x + \sqrt{x^2 - 1}\ \right)$ 5

(c) If $u = sin\ ^{-1}\left(\frac{x^{1/4} - y^{1/4}}{x^{1/5} - y^{1/5}}\right)$ then find the value of: $x^2 u_{xx} + 2xy\ u_{xy} + y^2 u_{yy}$ 5

CODE NO. : Z-1101-2015

(Revised Course)

FACULTY OF ENGINEERING AND TECHNOLOGY

F.Y.B.Tech. (All) EXAMINATION

MAY/JUNE, 2015

ENGINEERING MATHEMATICS-I

Time-Three Hours Maximum Marks-80

‘‘Please check whether you have got the right question paper.’’

N.B. :- (i) Question No.1 and Question No.6 are compulsory.

(ii) From section A solve any two section questions from the remaining Q.No.2,3,4 and 5

(iii) From section B solve any two section questions from the remaining Q.No.7,8,9 and 10

(iv) Figures to the right indicate full marks.

(v) Assume suitable data if necessary.

Section A

1. Solve any five (each for two marks) 10

(a) Define linear dependence and linear independence of vectors.

(b) Characteristic equation of matrix: $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ is ……………..

(c) Cayley-Hamilton theorem states that ……………….

(d) If $y = e^x sinx$ then $y_n =$ ………..

(e) Discus the convergence of p-series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$

(f) The Maclaurin’s theorem states that ………….

(g) Ratio test states that …………..

(h) The series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ is ……………

2. (a) Find the rank of matrix by reducing it to its normal form :

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$ 5

(b) If $x = \sin\left\{\frac{1}{a} Logy\right\}$ then show that: $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n$ 5

(c) Test the convergence of the series: $\frac{1}{1\,.2\,.3} + \frac{3}{2\,.3\,.4} + \frac{5}{3\,.4\,.5} + \cdots \infty$ 5

3. (a) Test for consistency and solve:

$$x_1 + 2x_2 - 3x_4 = 0$$

$$2x_1 - x_2 + x_3 + 7x_4 = 0$$

$$4x_1 + 3x_2 + 2x_3 + 2x_4 = 0$$ 5

(b) Evaluate: $\lim_{x\to\pi/2} [cosecx]^{tan^2 x}$ 5

(c) Examine the convergence of: $\sum_{n=2}^{\infty} \frac{1}{n\,Logn}$ 5

4. (a) Find Eigen values and Eigen vector corresponding to highest eigen value of matrix:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$ 5

(b) Prove that: $e^{a\sin^{-1}x} = 1 + ax + \frac{a^2x^2}{2!} + \frac{a(a^2+1)x^3}{3!} + \cdots$ 5

(c) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ 5

5. (a) Verify Cayley-Hamilton theorem for given matrix and hence find A^{-1}

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$ 5

(b) Find nth differential coefficient of: $y = \frac{x}{(x-1)(x-2)(x-3)}$ 5

(c) Test the convergence of the series: $1 + 3x + 5x^2 + 7x^3 + \cdots$ 5

Section B

6. Solve any five (each for two marks) 10

(a) Express $1 - i$ in polar form.

(b) Separate into real and imaginary part of $\sin(x + iy)$

(c) Find general value of $Logi$

(d) If $u = x(1 - y), v = xy$ then find the value of the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$

(e) If $u = x^2 + 2xy + y^2 + x + y$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \ldots\ldots..$

(f) If $u = x^3 + y^3$ where $x = acost, y = b\ sint$ find $\frac{du}{dt}$

(g) If u and v are functions of r and s where r, s are functions of x and y then

$\frac{\partial(u,v)}{\partial(r,s)}\ \frac{\partial(r,s)}{\partial(x,y)} = \ldots\ldots\ldots$

(h) If $u = Log(x^2 + y^2)$ then find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$

7. (a) Express $sin^7\theta$ as a sum of sines of multiples of θ. 5

(b) Verify Euler's theorem for function: $u = x^n \sin\left(\frac{y}{x}\right)$ 5

(c) Locate stationary points of: $x^4 + y^4 - 2x + 4xy - 2y^2$ and determine nature of function of these points. 5

8. (a) Prove that: $Log\left[\frac{\cos(x-iy)}{\cos(x+iy)}\right] = 2i\ tan^{-1}[tanx\ tanhy]$ 5

(b) If $u = lx + my, v = mx - ly$ then show that:

(i) $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{l^2}{l^2+m^2}$

(ii) $\left(\frac{\partial y}{\partial v}\right)_u \left(\frac{\partial v}{\partial y}\right)_x = \frac{l^2}{l^2+m^2}$ 5

(c) If $x = u(1-v), y = uv$ then prove that $JJ' = 1$ 5

9. (a) If $\cosh(\alpha + i\beta) = x + iy$ prove that

(i) $\frac{x^2}{cosh^2\alpha} + \frac{y^2}{sinh^2\alpha} = 1$

(ii) $\frac{x^2}{sin^2\beta} - \frac{y^2}{cos^2\beta} = 1$ 5

(b) If $z = xy^2 + x^2y, x = at^2, y = 2at$ then find $\frac{dz}{dt}$ 5

(c) Show that: $(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} cos\frac{n\pi}{4}$ 5

10. (a) If $|z+i| = |z|$ and amp$\left(\frac{z+i}{z}\right) = \frac{\pi}{4}$ find z 5

(b) If $u = Log(tanx + tany + tanz)$ then prove that

$sin2x\,\frac{\partial u}{\partial x} + sin2y\,\frac{\partial u}{\partial y} + sin2z\,\frac{\partial u}{\partial z} = 2$ 5

(c) If $cos\alpha + cos\beta + cos\gamma = 0, sin\alpha + sin\beta + sin\gamma = 0$ then prove that

(i) $cos3\alpha + cos3\beta + cos3\gamma = 3\cos(\alpha+\beta+\gamma)$

(ii) $sin3\alpha + sin3\beta + sin3\gamma = 3\sin(\alpha+\beta+\gamma)$ 5

Subject Code: 1204-2015

(Revised Course)

FACULTY OF ENGINEERING AND TECHNOLOGY

F.Y.B.Tech.(All) EXAMINATION

NOVEMBER/DECEMBER, 2015

ENGINEERING MATHEMATICS-I

Time-Three Hours   Maximum Marks-80

''Please check whether you have got the right question paper.''

N.B. :- (i) Question No.1 and Question No.6 are compulsory.

(ii) From section A solve any two section questions from the remaining Q.No.2,3,4 and 5

(iii) From section B solve any two section questions from the remaining Q.No.7,8,9 and 10

(iv) Figures to the right indicate full marks.

(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks)   10

(a) Define Rank of Matrix.

(b) If $y = \sin(2x + 3)$ then $y_n =$ ……..

(c) Cayley-Hamilton theorem sates that ………..

(d) Comparison test states that …………..

(e) Eigen values for the matrix $\begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}$ are …………..

(f) Cauchy's n^{th} root test states that …….. …

(g) Expansion of $f(x) = e^x$ in Maclaurin's series is ……….

(h) The Taylor's series theorem states that $f(x+h) = \ldots\ldots\ldots\ldots$

2. (a) Find Eigen values and Eigen vector for least Eigen value of matrix:

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$ 5

(b) If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$ then show that

(i) $(1-x^2)\, y_2 - 3xy_1 - y = 0$

(ii) $(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 = 0$ 5

(c) Test for convergence of the series:

$$\frac{2}{1} + \frac{3}{8} + \frac{4}{27} + \cdots + \frac{n+1}{n^2} + \cdots$$ 5

3. (a) Verify Caley-Hamilton theorem for the following matrix and hence find A^{-1}

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ 5

(b) Use taylor's theorem to find approximate value of $\sqrt{9.12}$ to four decimal places. 5

(c) Test for convergence of the following series:

$$\frac{1^3}{3} + \frac{2^3}{3^2} + \frac{3^3}{3^3} + \cdots$$ 5

4. (a) Discuss consistency of system of equations and solve

$$4x - 2y + 6z = 8$$

$$x + y - 3z = 1$$

$$15x - 3y + 9z = 21$$ 5

(b) Evaluate: $\lim_{x\to 0} \frac{\log(tanx)}{\text{logx}}$ 5

(c) Test the convergence of the series: $1 + \frac{2^2}{2!} + \frac{2^2}{2!} + \frac{2^2}{2!} + \cdots$ 5

5. (a) Examine for linear dependence or independence of the following vectors. If dependent find relation between them

$x_1 = (3,\ 1, -4),\ x_2 = (2,\ 2, -3)\ ,\ x_3 = (0, -4,\ 1)$ 5

(b) Expand: $(1+x)^{1/x}$ up to the term containing x^2 5

(c) Test for convergence of the series whose nth term is: $u_n = \frac{2^n}{n^3+1}$ 5

Section B

6. Solve any five (Each for two marks) 10

(a) Find locus of z given by $|3z-1| = |z-3|$

(b) Find modulus and argument of $\frac{1+2i}{1-3i}$

(c) Real part of $\cosh(x+iy)$ is …………

(d) If $z = f(x,y)$ where $x = f_1(u,v)$, $y = f_2(u,v)$ then $\frac{\partial z}{\partial u}$ =……. and $\frac{\partial z}{\partial v}$ = …….

(e) If $u = x^y$ then $\frac{\partial u}{\partial x}$ = …………..

(f) $\frac{x^{1/2}+x^{1/2}}{x^{3/2}+2x^{3/2}}$ is homogeneous function of degree ……….

(g) Stationary points of function $x^3+y^3-3axy=0$

(h) If $x = r\,cos\theta$, $y = r\,sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}$ = ……….

7. (a) Show that:

$$\left[\frac{1+sin\alpha+i\,cos\alpha}{1+sin\alpha-i\,cos\alpha}\right]^n = \cos\left(\frac{n\pi}{2}-n\alpha\right)+i\sin\left(\frac{n\pi}{2}-n\alpha\right)$$ 5

(b) Discuss maxima and minima of $x^3+3xy^2-15x^2-15y^2+72x$ 5

8. (a) Find continued product of all values of $\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]^{3/4}$ 5

(b) If $u = cosx\,, v = sinx\ cosy\,, w = sinx\ siny\ sinz$ then find the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ 5

(c) If $u = f(r)$ where $r = \sqrt{x^2+y^2+z^2}$ then prove that

$u_{xx} + u_{yy} + u_{zz} = f''(r) + \frac{2}{r}\ f'(r)$ 5

9. (a) If $\tan(\alpha + i\beta) = x + iy$ then show that

(i) $x^2 + y^2 + 2x\ cot2\alpha = 1$

(ii) $x^2 + y^2 - 2y\ coth2\beta + 1 = 0$ 5

(b) If $u = sin^{-1}(x^2+y^2)^{1/5}$ then show that

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{2}{15}\ tanu\ [3tan^3u - 3]$ 5

(c) Considering only principal value prove that real part of

$\left(1+i\sqrt{3}\right)^{\left(1+i\sqrt{3}\right)}$ is $2e^{-\pi/\sqrt{3}}\cos\left(\frac{\pi}{3}+\sqrt{3}\log 2\right)$ 5

10. (a) Prove that: $sinh^{-1}(tanx) = \log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ 5

(b) If $z = f(x,y), x = e^u cosv\,, y = e^u sinv$ then show that

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-2u}\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$ 5

(c) Use De-Moivre's theorem to express $cos5\theta$ in terms of $cos\theta$ 5

Subject Code: 1101-2016

(Revised Course)

FACULTY OF ENGINEERING AND TECHNOLOGY

F.Y.B.Tech.(All) EXAMINATION

APRIL/MAY, 2016

ENGINEERING MATHEMATICS-I

Time-Three Hours Maximum Marks-80

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(iii) From section B solve any two section questions from the remaining Q.No.7, 8, 9 and 10

(iv) Figures to the right indicate full marks.

(v) Assume suitable data if necessary.

Section A

1. Solve any five (Each for two marks) 10

(a) Define Rank of Matrix.

(b) Find characteristic equation of the matrix: $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

(c) Find the Eigen values of the matrix: $A = \begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}$

(d) State the ratio test of series

(e) If $y = sin3x\ cos4x$ then $y_n =$ ………..

(f) Examine the convergence of the series: $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

(g) The Maclaurin's theorem state that ………………….

(h) $\lim_{x\to 0} \frac{sin^2 x}{x}$ is equal to ……….

2. (a) reduce the following matrix into canonical form and hence find rank of :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$ 5

(b) Test for convergence of the series: 5

$$\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \left(\frac{1.2.3.4}{3.5.7.9}\right)^2 + \cdots$$

(c) If $y = \frac{1}{x^2+a^2}$ find y_n 5

3. (a) Test for consistency and solve: 5

$$4x - 2y + 6z = 8$$

$$x + y - 3z = -1$$

$$15x - 3y + 9z = 21$$

(b) Test for convergence of the following series: $\sum \frac{4.7 \ldots (3n+1)x^n}{n!}$ 5

(c) Find nth derivative of: $y = e^{2x} sin4x\ cos2x$ 5

4. (a) Find eigen values and eigen vector corresponding to smallest positive eigen value of the

matrix: $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ 5

(b) State the value of x for which the series converges: 5

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \frac{5^5 x^5}{5!} + \cdots$$

(c) Expand $tan^{-1} x$ in powers of $(x - 1)$ 5

5. (a) Investigate the values of λ and μ so that equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$x + 2y + \lambda z = \mu$ has

(i) No solution

(ii) Unique Solution

(b) Test the series for absolute or conditional convergence:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$$ 5

(c) Evaluate: $\lim_{x \to 1} \left[\frac{1}{logx} - \frac{x}{x-1}\right]$ 5

Section B

6. Solve any five (Each for two marks) 10

(a) Write formula for DeMoivre's theorem.

(b) Prove that: $\frac{(cos\theta + i\ sin\theta)^m}{(sin\theta + i\ cos\theta)^n} = (-1)^n[\cos(m\theta + n\theta) + i\sin(m\theta + n\theta)]$

(c) Separate into real and imaginary part of $\cos(x + iy)$

(d) Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ for $u = x^4y^2\ sin^{-1}\left(\frac{y}{x}\right)$

(e) If $u = e^{x^4}$ then find $\frac{\partial^2 u}{\partial x \partial y}$

(f) Write a formula for total differentiation.

(g) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ if $u = x\ siny, v = x\ cosy$

(h) If u, v, w are continuous and dfferentiatble function of three independent variables x, y, z

then $\frac{\partial(u,v,w)}{\partial(x,y,z)} =$ ……………..

7. (a) Find the locus of point z such that: $|z + i| = |z|$ and amp$\left(\frac{z+1}{z}\right) = \frac{\pi}{4}$ 5

(b) If u =log$(x^3 + y^3 + z^3 - 3xyz)$ show that: $\left(\frac{\partial}{\partial x} + \frac{\partial}{yx} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ 5

(c) If $x = e^{\theta} cos\phi, y = e^{\theta} sin\phi$ then prove that: $JJ' = 1$ 5

8. (a) Prove that: $cos\,^8\theta = \frac{1}{28}[cos8\theta + 8\,cos6\theta + 56\,cos2\theta + 35]$ 5

(b) If $u = tan\,^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$ 5

(c) Examine the function for extreme values $x^2 + y^2 + 6x + 12$ 5

9. (a) Use DeMoivres theorem to solve : $x^9 - x^5 + x^4 - 1 = 0$ 5

(b) If $z = f(x, y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$ then show that

$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$ 5

(c) Prove that: $sinh\,^{-1}x = \log\left(x + \sqrt{x^2 + 1}\right)$ 5

10. (a) Prove that real part of $(1 + i\sqrt{3})^{\,1+i\sqrt{3}}$ is $2e^{-\frac{\pi}{\sqrt{3}}}$ cos$\left(\frac{\pi}{3} + \sqrt{3}\,Log2\right)$ 5

(b) Find $\frac{dy}{dx}$ when $(cosx)^y = (siny)^x$ 5

(c) If $\tan(x + iy) = \sin(u + iv)$ Prove that: $\frac{sin2x}{sinh2y} = \frac{tanu}{tanv}$ 5

Compiled & Collected by

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