Unit-I Linear Differential Equation

1. On putting $x = e^z$,the transformed differential equation of $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$ is ………. [B.Tech May/June 2013]
2. Solve: $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 1$ [B.Tech May/June 2013]
3. An e.m.f. $E\sin pt$ is applied at $t = 0$ to a circuit containing a condenser C and inductance L in series the current satisfies the equation: $L\frac{di}{dt} + \frac{1}{C}\int i\,dt = E\sin pt$ if $P^2 = \frac{1}{LC}$ and initially the current i and charge q are zero.show that the current at any time t is given by $\frac{E}{2L}$ t sinpt. [B.Tech May/June 2013]
4. Solve: $(D^2 + 4)y = 4\sec^2 2x$ [B.Tech May/June 2013]
5. Solve: $(D^2 - 1)y = \cosh x \cos x$ [B.Tech May/June 2013]
6. Differential equation of an electric ckt. containing an inductance L and capacitance C is ……….. [B.Tech(Old) May/June 2013]
7. Convert Legendre's differential equation into constant coefficient LDE for $(2x+3)^2\frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = \mathrm{Sin}(\lg(2x+3))$ [B.Tech(Old) May/June 2013]
8. Solve: $(D^4 - 1)y = \cosh x \cos x$ [B.Tech(Old) May/June 2013]
9. Solve by method of variation of parameter $(D^2 - 1)y = \frac{2}{1+e^x}$ [B.Tech(Old) May/June 2013]
10. An e.m.f. $E\sin pt$ is applied at $t = 0$ to a LC ckt. The current i satisfies the equation: $L\frac{di}{dt} + \frac{1}{C}\int i\,dt = E\sin pt$; $i = -\frac{dq}{dt}$, $P^2 = \frac{1}{LC}$. Initially $q = 0, i = 0$, find current i at any time t. [B.Tech(Old) May/June 2013]
11. Convert Cauchy's LDE to constant coefficient LDE for: $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} = x^4$ [B.Tech Nov/Dec 2012]
12. Solve: $(4x^2D^2 + 1)y = 19\cos(\log x) + 22\sin(\log x)$ [B.Tech Nov/Dec 2012]
13. A light horizontal strut AB is freely pinned at A and B. It is under the action of equal and opposite compressive forces P at its ends and it carries a load W at its centre.Then for : $0 < x < \frac{l}{2}$, $EI\frac{d^2y}{dx^2} + Py + \frac{1}{2}Wx = 0$. Also $y = 0$ at $x = 0$ and $\frac{dy}{dx} = 0$ at $x = \frac{l}{2}$. Prove that: $y = \frac{W}{2P}\left[\frac{\sin nx}{n\cos\frac{nl}{2}} - x\right]$ where $n^2 = \frac{P}{EI}$ [B.Tech Nov/Dec 2012]
14. Solve: $(D^3 - D^2 - 6D)y = 1 + x^2$ [B.Tech Nov/Dec 2012]
15. Solve: $(x^2D^2 - xD + 4)y = \cos(\log x) + x\sin(\log x)$ [B.Tech(old) Nov/Dec 2012]
16. Solve: $(D^2 + 3D + 2)y = e^{e^x}$ [B.Tech(old) Nov/Dec 2012]
17. Solve: $(1+x)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = (1+x)^4$ [B.Tech(old) Nov/Dec 2012]

18. Differential equation of an electric circuit containing an inductance L, capacitance C and resistance R is …….. [B.Tech April/May 2012]

19. Convert Cauchy's LDE to constant coefficient LDE for:
$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = \log x$ [B.Tech April/May 2012]

20. Solve: $(x^3D^3 + x^2D^2 - 2)y = x + \frac{1}{x^3}$ [B.Tech April/May 2012]

21. The deflection of a strut of length l with one end built and other end subjected to end thrust P satisfies the equation $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P}(l - x)$ Given that $y = \frac{dy}{dx} = 0$ when $x = 0$ and $y = 0$ when $x = l$. Find the deflection curve and show that $al = \tan al$.

22. Solve by the method of variation of parameters:
$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x)$ [B.Tech Nov/Dec 2009]

23. An electric circuit consists of an inductance L, a condenser of capacitance C, and e.m.f. $E = E_0 \cos\omega t$, so that the charge Q satisfies the differential equation $\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E_0}{L}\cos\omega t$ If $\omega = \frac{1}{\sqrt{LC}}$ and initially at $t = 0, Q = Q_0$ and current $i = i_0$. Find the charge at any time t.
[B.Tech Nov/Dec 2009]

24. Solve: $\frac{dx}{dt} + 2y = e^t$, $\frac{dy}{dt} - 2x = e^{-t}$ [B.Tech Nov/Dec 2009]

25. If $(D - a)y = \phi(x)$, then $\frac{1}{D-a}\phi(x) = \ldots\ldots$ [B.Tech(old) Nov/Dec 2009]

26. If P.I.= $u\,\phi_1(x) + v\,\phi_2(x)$ then $u = \ldots\ldots.$ [B.Tech(old) Nov/Dec 2009]

27. If $f(D)y = xv$, then $\frac{1}{f(D)}xv = \ldots\ldots\ldots$ [B.Tech(old) Nov/Dec 2009]

28. If $f(D)y = e^{ax}u$, then $\frac{1}{f(D)}e^{ax}u = \ldots\ldots\ldots$ [B.Tech(old) Nov/Dec 2009]

29. Solve by the method of variation of parameters:
$(D^3 + D)y = \operatorname{cosec} x$ [B.Tech(old) Nov/Dec 2009]

30. Solve: $(2x + 3)^2\frac{d^2y}{dx^2} - 2(2x + 3)\frac{dy}{dx} - 12y = 6x$ [B.Tech(old) Nov/Dec 2009]

31. Solve: $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$ Given $x = y = 0$ when $t = 0$
[B.Tech(old) Nov/Dec 2009]

32. Write down the differential equation for change in R-C circuit with e.m.f. 2 cosωt.
[B.Tech April/May 2010]

33. Use method of variation of parameter to solve:
$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ [B.Tech April/May 2010]

34. Solve simultaneous differential equation:
$\frac{dx}{dt} + 4x + 3y = t$, $\frac{dy}{dt} + 2x + 5y = e^t$ [B.Tech April/May 2010]

35. Get the solution of differential equation:
$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = \sin(\log x)$ [B.Tech April/May 2010]

36. If V is a function of x , then $\frac{1}{f(D)} x.V = \ldots\ldots$ [B.Tech April/May 2010]

37. $\frac{1}{D-k} X(x) = \ldots\ldots$ [B.Tech April/May 2011]

38. Explain the method to solve Cauchy's Linear equation. [B.Tech April/May 2011]

39. The differential equation of L-C-R circuit (without emf) is…. [B.Tech April/May 2011]

40. An emf E sinωt is applied to L-R circuit , find the current i in terms of time t. [B.Tech April/May 2011]

41. Solve the simultaneous equations:
$\frac{dx}{dt} + 2x + 3y = 0,\ \frac{dy}{dt} + 3x + 2y = 0$ [B.Tech April/May 2011]

42. Solve: $x\frac{dy}{dx} + y = \log x$ [B.Tech April/May 2011]

43. Solve: $(1+2x)^2\frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$ [B.Tech April/May 2011]

44. Solve: $(D^2 + a^2)y = \tan ax$ [B.Tech April/May 2011]

45. The differential equation of free oscillations (without force) is given by …….

46. Obtain the solution of
$x^3y''' + 2x^2y'' + 2y = 10(x + \frac{1}{x})$ [B.Tech Nov/Dec 2010]

47. Use variation of parameters to solve:
$\frac{d^2y}{dx^2} + y = x\sin x$ [B.Tech Nov/Dec 2010]

48. Solve: $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ [B.Tech Nov/Dec 2010]

49. Solve: $\frac{dx}{dt} = 7x - y,\ \frac{dy}{dt} = 2x + 5y$ [B.Tech Nov/Dec 2010]

50. Differential equation of L-C-R circuit with emf V is ……. [B.Tech Nov/Dec 2011]

51. $\frac{1}{(D^2+a^2)^2}\sin(ax+b) = \ldots\ldots$ [B.Tech Nov/Dec 2011]

52. Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ [B.Tech Nov/Dec 2011]

53. The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$. Find i as a function of t, given $i = 0$ at $t = 0$.
[B.Tech Nov/Dec 2011]

54. Solve: $(x^2D^2 + 3xD + 10)y = 9x^2$ [B.Tech Nov/Dec 2011]

55. Solve by using method of variation of parameters:
$(D^2 + a^2)y = \sec ax$ [F.E. May/June 2008]

56. A horizontal simply supported beam of length l bends under its own weight w kg/m satisfies $EI\frac{d^2y}{dx^2} = \frac{wx^2}{2} - \frac{wlx}{2}$ where y is deflection. Find maximum deflection.
[F.E. May/June 2008]

57. Solve: $(2x+1)^2\frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x$ [F.E. May/June 2008]

58. Solve: $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2\log x$ [F.E. May/June 2008]

59. An electric circuit consists of an inductance L, a condenser of capacitance C, and e.m.f.

$E = E_0 \cos\omega t$, so that the charge Q satisfies the differential equation $\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E_0}{L}\cos\omega t$
If $\omega = \frac{1}{\sqrt{LC}}$ and initially at $t = 0, Q = Q_0$ and current $i = i_0$. Find the charge at any time t.
[F.E. May/June 2008]

60. Solve: $[D^2 - 1]y = x^2 \sin 3x$ [F.E. May/June 2008]
61. Solve by using method of variation of parameters:
$(D^2 - 1)y = (1 + e^{-x})^2$ [F.E. Nov/Dec 2006]
62. An uncharged condenser of capacity C is charged by applying an e.m.f of values:
$E \sin\left(\frac{t}{\sqrt{LC}}\right)$ through leads of inductance L of negligible resistance. The charge q on the plate of condenser satisfies the equation: $\frac{d^2q}{dt^2} + \frac{1}{LC}q = \frac{E}{L}\sin\left(\frac{t}{\sqrt{LC}}\right)$ Prove that the charge at any time t is : $\frac{EC}{2}\left[\sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{t}{\sqrt{LC}}\cos\left(\frac{t}{\sqrt{LC}}\right)\right]$ [F.E. Nov/Dec 2006]
63. Solve: $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$ [F.E. Nov/Dec 2006]
64. Solve: $(2x + 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} + \frac{y}{2x+1} = \frac{3x+4}{2x+1}$ [F.E. Nov/Dec 2006]
65. Solve by using method of variation of parameters:
$(D^3 + D)y = \text{cosec}\, x$ [F.E. Nov/Dec 2006]
66. Solve: $(1 + x)^2\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 2\sin[10y(1 + x)]$ [F.E. May/June 2006]
67. Without using variation of parameter method solve:
$(D^2 + 3D + 2)y = e^{e^x}$ [F.E. May/June 2006]
68. Solve: $\frac{dx}{dt} - \frac{dy}{dt} + 2y = \cos 2t$, $\frac{dx}{dt} + \frac{dy}{dt} - 2x = \sin 2t$ [F.E. May/June 2006]
69. The deflection of a strut of length l with one end built in and the other supported and subjected to end thrusrt P satisfies the equation: $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P}(l - x)$
Prove that the deflection is $y = \frac{R}{P}\left[\frac{\sin ax}{a} - l\cos ax + l - x\right]$ where $al = \tan al$.
70. Solve: $(D^2 - 4D + 4)y = \frac{e^{2x}}{x}$ by variation of parameter method. [F.E. May/June 2006]
71. Solve: $[D^2 + 1]y = \tan x$ [F.E. May/June 2006]
72. Solve the equation: $EI\frac{d^2y}{dx^2} + Py = -\frac{wl^2}{8}\sin\frac{\pi}{l}x$ for a strut of length l freely hinged at each end. Prove that the deflection y at the centre is :
$\frac{wl^2}{8(Q-P)}$ where $Q = \frac{EI\pi^2}{l^2}$ [F.E. May/June 2006]
73. Solve: $(x + b)\frac{d^2y}{dx^2} - 4(x + b)\frac{dy}{dx} + 6y = x$ [F.E. May/June 2006]
74. A light horizontal strut of length l is freely pinned at both the ends. It is under the action and opposite compressive forces p at its ends and it carries a load w at its centre.Then for $0 < x < \frac{l}{2}$, $EI\frac{d^2y}{dx^2} + py + \frac{1}{2}wx = 0$ also $y = 0$ at $x = 0$ and $\frac{dy}{dx} = 0$ at $x = \frac{l}{2}$ Prove that

$y = \frac{w}{2p}\left[\frac{\sin nx}{n\cos\left(\frac{nl}{2}\right)} - x\right]$ where $n^2 = \frac{p}{EI}$

75. Differential equation of an electric ckt containing an inductance L, capacitance C and Resistance R (without battery) is …… [May/June 2015]

76. $\frac{1}{D-k}X(x) = \ldots\ldots.$ where $X(x)$ is a function of x [May/June 2015]

77. The C.F. of the equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = x^2$ is ………. [May/June 2015]

78. If $f(b) = 0$ then $\frac{1}{f(D)}\, e^{bx} = \ldots\ldots\ldots\ldots$ [May/June 2015]

79. Solve: $(D^2 - 4D + 3)y = e^x\cos 2x$ [May/June 2015]

80. Solve: $(D^2 + a^2)y = \operatorname{cosec} ax$ by general method. [May/June 2015]

81. Solve: $(2x+1)^2\frac{d^2y}{dx^2} - (2x+1)\frac{dy}{dx} + y = 3x + 4$ [May/June 2015]

82. An uncharged condenser of capacity C is charged by applying an e.m.f of values:

$E\sin\left(\frac{t}{\sqrt{LC}}\right)$ through lead of self inductance L.t gives by the differential equation:

$\frac{d^2q}{dt^2} + \frac{1}{LC}q = \frac{E}{L}\sin\left(\frac{t}{\sqrt{LC}}\right)$ Prove that the charge at any time t is:

$\frac{EC}{2}\left[\sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{t}{\sqrt{LC}}\cos\left(\frac{t}{\sqrt{LC}}\right)\right]$

83. A strut of length l freely hinged at each end, satisfies the equation :

$EI\frac{d^2y}{dx^2} + Py = -\frac{wl^2}{8}\sin\left(\frac{\pi}{l}x\right)$ Prove that, the deflection at the centre is $\frac{wl^2}{8(Q-P)}$ where

$Q = \frac{EI\pi^2}{l^2}$

84. Solve by method of variation of parameter: $(D^3 + D)y = \operatorname{cosec} x$

Unit-II Vector Differentiation

1. If $\bar{r} = x\,i + yj + zk$ and $= |\bar{r}|$, $\text{div}\left(\frac{\bar{r}}{|\bar{r}|^3}\right) =$………. [B.Tech.May/June 2013]
2. The value of λ so that the vector $\bar{F} = (x + 5y)i + (y - 3z)j + (\lambda z + x)k$ is solenoidal is ……… [B.Tech.May/June 2013]
3. Find the radial and transverse components of velocity and acceleration of the particle moving along the curve $r = ae^{b\theta}$ with constant angular velocity.
4. Find the direction derivative of $\phi = 4e^{2x-y+z}$ at $(1,\ 1, -1)$ in the direction towards the point $(-3,5,6)$. [B.Tech.May/June 2013]

   Show that $\bar{F} = (y\sin z - \sin x)i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$ is irrotational(or conservative).Find the corresponding scalar function ϕ such that $\bar{F} = \nabla\phi$. [B.Tech.May/June 2013]
5. Write definition of vector point function. [B.Tech Nov/Dec 2012]
6. Radial component of acceleration is………… [B.Tech Nov/Dec 2012]
7. A particle moves along a curve at :
8. $\bar{r} = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$. Find the tangential and normal components of acceleration at $t = 2$. [B.Tech Nov/Dec 2012]
9. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -2,\ 2)$ in the direction $\nabla(x^2y^2z^2)$ at $(1, -1,\ 1)$. [B.Tech Nov/Dec 2012]
10. Find the angle between the surfaces: $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1,\ 2)$. [B.Tech Nov/Dec 2012]
11. If $\bar{r} = xi + yj + zk$ then $\text{curl}\bar{r} =$…… [B.Tech (Old)Nov/Dec 2013]
12. If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$ then $\text{grad}(\bar{a}.\bar{r}) =$……..
13. Find the directional derivative of scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point $A(1, -1,\ 1)$ in the direction of the line AB, where B has coordinates $(3,21)$. [B.Tech (Old)Nov/Dec 2013]
14. Show that $\bar{F} = (2xy + z^3)i + x^2j + 3z^2x\,k$ is conservative field and find scalar potential. [B.Tech (Old)Nov/Dec 2013]
15. If $\bar{v} = e^{xyz}\ (i + j + k)$ then $\nabla \times \bar{v}$ at the point $(1,2,3)$ is …….. [B.Tech (Old)Nov/Dec 2012]
16. Find the directional derivative of $\phi = e^{2x}\cos yz$ at the origin in the direction of the tangent to the curve $x = a\sin t, y = a\cos t, z = at$ at $= \frac{\pi}{4}$.

    Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational. [B.Tech (Old)Nov/Dec 2012]
17. $\nabla.\left(\bar{A} \times \bar{B}\right) =$ ………… [B.Tech (Old)Nov/Dec 2012]
18. If $\bar{F}$ is such that $\nabla.\bar{F} = 0$, then $\bar{F}$ is …………… [B.Tech (Old)Nov/Dec 2012]
19. $\nabla\left(f\,\bar{A}\right) =$………… [B.Tech (Old)May/June 2012]

20. If $\overline{F}$ is such that $\nabla \times \overline{F} = 0$, then $\overline{F}$ is …………… [B.Tech (Old)May/June 2012]
21. Determine the constant b such that
$\overline{A} = (bx + 4y^2z)i + (x^3\sin z - 3y)j - (e^x + 4\cos x^2\, y)k$ is solenoidal.
22. Find the directional derivative of $\phi = x^2yz^2$ along the curve $x = e^{-t}$,
$y = 2\sin t + 1, z = t - \cos at$ at $t = 0$. [B.Tech (Old)May/June 2012]
23. Determine the constants a and b such that curl of :
$(2xy + 3yz)i + (x^2 + axz - 4z^2)j + (3xy + 2byz)k$ is zero.
24. Find the tangential and normal components of acceleration at any time t of a particle whose position vector at any time t is given by: $x = e^t\cos t, y = e^t\sin t$.
25. If ϕ_1 and ϕ_2 are two scalar functions then $\nabla(\phi_1\phi_2) =$……….. [B.Tech 2011]
26. If $\overline{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ then $\nabla \times \overline{F} =$………[B.Tech 2011]
27. $\nabla(\phi\, \overline{A}) =$………… [F.E. 2011]
28. Find unit tangent vector at any point on the curve $x = 3\cos t, y = 3\sin t, z = 4t$.
29. If $\overline{F} = (x + y + 1)i + j - (x + y)k$ show that $\overline{F} \cdot \mathrm{curl}\overline{F} = 0$ [B.Tech 2011]
30. If $\overline{r} = xi + yj + zk$ then $\nabla r =$………… [B.Tech 2011]
31. $\mathrm{curl}(\mathrm{grad}\phi) =$……… [B.Tech 2011]
32. If $\overline{F}(t)$ has constant magnitude then $\overline{F}.\frac{d\overline{F}}{dt} =$………. and $\overline{F}(t)$ has constant direction then $\overline{F} \times \frac{d\overline{F}}{dt} =$……….[B.Tech 2010]
33. For a constant vector $\overline{a}$, $\mathrm{div}\overline{a} =$…….. , $\mathrm{curl}\overline{a} =$……… [B.Tech 2010]
34. If $\overline{r} = xi + yj + zk$ then $\nabla r^n =$………… [F.E. 2010]
35. If ϕ_1 and ϕ_2 are two scalar functions then $\nabla\left(\frac{\phi_1}{\phi_2}\right) =$……….[B.Tech 2010]
36. If $\overline{r} = xi + yj + zk$ then $\nabla\left(\frac{1}{r}\right) =$…… [B.Tech 2010]
37. $\mathrm{curl}(\mathrm{curl}v) =$……..
38. Find the radial and transverse components of acceleration at $t = \pi$ for the curve
$r = 2 + 3\cos\theta$ and $\theta = \sin t$. [B.Tech 2010]
39. Trnsverse components of acceleration=……….[B.Tech 2009]
40. $\mathrm{grad}(\mathrm{Log} r) =$……….. [B.Tech 2009]
41. $\mathrm{div}(r^n\overline{r}) =$…………..[B.Tech 2009]
42. $\mathrm{curl}\left(\frac{1}{r}\sin r\right) =$…….. [B.Tech 2009]
43. A particle describes the curve $r = 2a\cos\theta$ with constant angular speed. Find the radial and transverse components of velocity and acceleration. [B.Tech 2009]
44. For the curve $x = t^3 + 1, y = t^2, z = t$ find the magnitude of tangential and normal components of acceleration for a particle moving on the curve at $t = 2$.
45. If $\overline{F} = \mathrm{grad}(x^2 + y^2 + z^2 - 2xyz)$ then find $\mathrm{div}\overline{F}$ [F.E. 2013]
46. Show that : $\overline{F} = (y^2\cos x + z^3)i + (2y\sin x - 4)j + (2xz^2 + 2)k$ is irrotational.
47. Prove that $r^n\,\overline{r}$ is solenoidal only if $n = -3$. [F.E. 2013]

48. Find the directional derivative of the scalar function $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$in the direction of the normal to the surface $x(\log z) - y^2 = -4$ at the point $(-1, 2, 1)$. [F.E. 2013]
49. Prove that: $\nabla^2 r^n = n(n+1)r^{n-2}$ [F.E. 2013]
50. Find $\nabla . \overline{A}$ if : $\overline{A} = \frac{xi + yj + zk}{\sqrt{x^2+y^2+z^2}}$ [F.E. 2013]
51. Find the tangential and normal component of acceleration for the curve $x = \text{Log}(t^2 + 1), y = t - 2\tan^{-1} t$ [F.E. 2013]
    Find the radial and transverse components of acceleration of a particle moving with constant angular velocity ω along the curve $r = a(1 - \cos\theta)$. [F.E. 2013]
52. A particle describes the straight line $x = a$ with constant angular velocity ω.Find radial and transverse components of acceleration. [F.E.May/June 2008]
53. Find the directional derivative of $x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at $(1,2,3)$. [F.E.May/June 2008]
54. A vector $\overline{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational.Find a, b, c. [F.E.May/June 2008]
55. Is the vector $\overline{F} = (\sin y + z)i + (x\cos y - z)j + (x - y)k$ is irrotational?[May/June2015
56. $\nabla r^n = \ldots\ldots\ldots\ldots\ldots\ldots$
57. If $\overline{r}$ is the position vector, prove that: $\nabla^2 r^n = n(n+1)r^{n-2}$
58. Find the directional derivative of the function, $\phi = x^2y + xyz + z^3$ at $(1, 2, -1)$ along the normal to the surface $x^2y^3 = 4xy + y^2z$ at $(1, 2, 0)$
59. Show that the vector field: $\overline{F} = 2xye^z i + x^2e^z j + x^2ye^z k$ is an irrotational vector field and hence find corresponding scalar potential such that $\overline{F} = \nabla\phi$
60. A particle moves in a plane with constant angular velocity ω about O. If the rate of increase of acceleration is totally (wholely) radial prove that : $\frac{d^2r}{dt^2} = \frac{1}{3} r\omega^2$

Unit-III Statistics

1. Calculate the coefficient of variation for the following data:

Class	1-10	11-20	21-30	31-40	41-50	51-60
f	3	16	26	31	16	8

2. Find the standard deviation and coefficient of variation of the following data:

No.of Childrens	0	1	2	3	4	5	6	7
No.of families	171	74	50	25	13	7	6	4

3. Find S.D. and mean deviation from the following data:

Class:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency:	8	12	17	14	9	7	4

Ans: Mean = 30.78 , S.D. = 16.64 , Mean Deviation = 13.906

4. What is S.D., variance, coefficient of variation for the data given below?

Marks:	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No.of Students:	6	8	17	21	15	11	2

Ans: S.D. = 7.48, C.V. = 0.64

5. What will be mean deviation and coefficient of mean deviation for the data

Marks:	32	37	42	47	52	57
No.of Students:	5	10	15	30	5	5

Ans: Mean = 44.5 , mean deviation = 5 , coefficient of mean deviation = 0.1124

6. If the given data is

Wages (in rupees) per days:	219	216	213	210	207	204	201	198	195
No. of persons:	2	4	6	10	11	7	5	4	1

Find mean, median, mode

Ans: mean = 207.5, median = 207 , mode = 207

7. Find karl pearson's coefficient and coefficient of skewness based on quartiles for the data

Wages (less than in rupees):	10	20	30	40	50
No.of person's:	5	15	35	40	50

8. What is lower quartile and upper quartile for the following data:

Values:	4.5	5	5.5	6	6.5	7	7.5	8
No. of students:	1	2	4	5	15	30	60	95

Ans: $Q_1 = 7$, $Q_3 = 8$

9. Find β_2 for the following distribution and comment on the nature of β_2

Class	1-3	3-5	5-7	7-9	9-11	11-13	13-15
f	13	9	9	8	15	12	18

Ans: Mean = 8.6 , $\mu_2 = 18.02$, $\mu_4 = 549$, $\beta_2 = 1.7 < 3$ it is platykurtic

10. Find the four central moments for the following information:

x_i	2	2.5	3	3.5	4	4.5	5
f_i	5	38	65	92	70	40	10

Also find β_1 and β_2, state the nature of β_2

Ans: Mean = 3.54 , $\mu_1 = 0$, $\mu_2 = 1.8$, $\mu_3 = 0.08$, $\mu_4 = 8.03$,

$\beta_1 = 0.001, \beta_2 = 2.44 < 3$, It is platykurtic

11. Find the first four central moments for the following data:

Class	10-19	20-29	30-39	40-49	50-59	60-69
f	12	20	30	15	10	5

Unit-IV Laplace Transform

1. If $f(t+4) = f(t)$ then formula for will be given by $L[f(t)] = \ldots$[B.Tech May 2013]
2. $L(t\,\text{sin}at) = \ldots\ldots.$ [B.Tech May/June 2013]
3. Find Laplace transform of $f(t) = \frac{1-e^{-t}}{t}$ [B.Tech May/June 2013]
4. Find: $L\{\int_0^t \sin(2t+3)\,dt\}$ [B.Tech May/June 2013]
5. Find: $L\{t^2 \cos at\}$ [B.Tech May/June 2013]
6. Find Laplace transform of
   $f(t) = k, \ 0 < t < a$
   $= -k, \ a < t < 2a$ and $f(t+2) = f(t)$ [B.Tech May/June 2013]
7. If $L\{f(t)\} = \bar{f}(s)$ and
   $F(t) = f(t-a), \ t > a$
   $= 0, \ t < a$
   then $L\{F(t)\} = \ldots\ldots\ldots$ [B.Tech(Old) May/June 2013]
8. $L\{f(t-a) \ U(t-a)\} = \ldots\ldots.$ [B.Tech(Old) May/June 2013]
9. Find $L\{F(t)\}$ for
   $F(t) = (t-1)^2, \ t > 1$
   $= 0, \ t < 1.$ [B.Tech(Old) May/June 2013]

10. Find: $L\{\int_0^t t\, e^{-4t} \sin 3t\, dt\}$ [B.Tech(Old) May/June 2013]

11. Express into Heaviside function and hence find Laplace transform of

$f(t) = \sin t,\ 0 < t < \pi$

$= \cos t, t \geq \pi$ [B.Tech(Old) May/June 2013]

12. If $L\{f(t)\} = \text{Log}\left(\frac{s+3}{s+1}\right)$ then $L\{f(2t)\} =$…… [B.Tech Nov/Dec 2012]

13. If $L\{f(t)\} = F(s)$ then $L\{t^n f(t)\} =$ ……… [B.Tech Nov/Dec 2012]

14. Find $L\{(\sin 2t - \cos 2t)^2\}$ [B.Tech Nov/Dec 2012]

15. Find Laplace transform of the function (Half wave rectifier)

$f(t) = \sin \omega t, \text{for } 0 < t < \frac{\pi}{\omega}$

$= 0 \quad \text{for} \quad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$ [B.Tech Nov/Dec 2012]

16. Find: $L\left\{\frac{e^{-t}\sin t}{t}\right\}$ [B.Tech Nov/Dec 2012]

17. Evaluate: $\int_0^\infty e^{-4t} \cosh^3 t\ dt$ [B.Tech Nov/Dec 2012]

18. Find Laplace transform of : $(t+1)^2 e^t$ [B.Tech(Old) Nov/Dec 2012]

19. Find: $L\{\sinh t \int_0^t t^2 e^{-t}\, dt\}$ [B.Tech(Old) Nov/Dec 2012]

20. $L\{t^2 - e^{-2t} + \cosh 2t\} =$…….. [B.Tech May/June 2012]

21. If $L\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$ then $L\{J_0(3t)\} =$…… [B.Tech May/June 2012]

22. $L\{t\, u(t-4) + t^2\, \delta(t-4)\} =$……… [B.Tech May/June 2012]

23. Prove that: $\int_0^\infty e^{-2t}\, t^2 \sin 3t\, dt = \frac{18}{2197}$ [B.Tech May/June 2012]

24. Find the L.T. of $t^2\ u(t-2) - \cosh t\, \delta(t-2)$ [B.Tech May/June 2012]

25. Find Laplace transform of $t^{3/2} + e^{-t}(\cos t + t^2)$ [B.Tech Nov/Dec 2009]

26. Find Laplace transform of $\int_0^t t\, e^{2t} \sin 4t\, dt$ [B.Tech Nov/Dec 2009]

27. If $L\{f(t)\} = F(s)$ then $L\left\{\frac{f(t)}{t}\right\} =$…….. [B.Tech(Old) Nov/Dec 2009]

28. Find the Laplace transform of : $t\, e^{-4t} \sin 3t$ [B.Tech(Old) Nov/Dec 2009]

29. Express into Heaviside function and hence find Laplace transform of

$f(t) = t^2,\ 0 < t < 1$

$= 4t\ \ , t > 1$ [B.Tech(Old) Nov/Dec 2009]

30. Find $L\{t^2 e^{-3t}\}$ [B.Tech(Old) Nov/Dec 2009]

31. If f(t) is continuous function and $L[f(t)] = \bar{f}(s)$ then $L[f''(t)] =$……….

32. $L\{f(t-a)\ u(t-a)\} =$ ……. [B.Tech May/June 2010]

33. Find the Laplace transform of : $\sin at - at \cos at$ [B.Tech May/June 2010]

34. Find the Laplace transform of : $e^{-3t}\, u(t-2)$ [B.Tech May/June 2010]

35. Find $L\left[\frac{\sinh t}{t}\right]$ [B.Tech May/June 2010]

36. If $f(t+T) = f(t)$ then $L[f(t)] =$…….. [B.Tech May/June 2011]

37. $L[\delta(t)] =$……… [B.Tech May/June 2011]

38. Find: $L[te^{at} \sin at]$ [B.Tech May/June 2011]

39. Find the Laplace transform of : $(t-1)^{2}u(t-1)$ [B.Tech May/June 2011]

40. Find: $L[t.\sin^{2}3t]$ [B.Tech May/June 2011]

41. If $L[f(t)] = \bar{f}(s)$ then $L[\int_0^t f(t)dt] = \ldots\ldots\ldots$ [B.Tech(Old) May/June 2010]

42. If $L[f(t)] = \bar{f}(s)$ then $L[f(at)] = \ldots\ldots.$ [B.Tech(Old) May/June 2010]

43. $L[u(t-a)] = \ldots\ldots$ where $u(t-a)$ is step function. [B.Tech(Old) May/June 2010]

44. Find the Laplace transform of : t^2 sinat [B.Tech(Old) May/June 2010]

45. Find: $L[\text{sint}.u(t-\pi)]$ [B.Tech(Old) May/June 2010]

46. Find the Laplace transform of : $\frac{\text{cosat} - \text{cosbt}}{t}$ [B.Tech(Old) May/June 2010]

47. $L\left[\frac{t}{2a}.\text{sinat}\right] = \ldots\ldots.$ [B.Tech Nov/Dec 2011]

48. If: $u(t-a) = 0, t < a$

$= 1, t \geq a$

Then $L[f(t-a)u(t-a)] = \ldots\ldots.$ [B.Tech Nov/Dec 2011]

49. Find the periodic function of period l by using figure:

50. Evaluate: $\int_0^\infty e^{-2t}\ \frac{\text{sinht}}{t}\ dt$ [F.E.(Old) May/June 2013]

51. Express F(t) in terms of Heaviside unit step function and hence find its Laplace transform

$F(t) = \text{cost},\ 0 < t < \pi$

$= \text{cos2t}, \pi < t < 2\pi$

$= \text{cos3t}, t > 2\pi$ [F.E.(Old) May/June 2013]

52. Find the Laplace transform of : $t\,e^{2t} - \frac{2\sin 3t}{t}$ [F.E.(Old) May/June 2013]

53. Find the Laplace transform of the sawtooth wave function defined by:

$F(t) = \frac{kt}{T}\ ,\ 0 < t < T$ and $f(t+T) = f(t)$ [F.E.(Old) May/June 2013]

54. Evaluate: $\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\ dt$ [F.E. May/June 2008]

55. Find the Laplace transform of

$F(t) = e^{-3(t-4)}\ \sin 4(t-4)$ for $t > 4$

$= 0$ for $t < 4$ [F.E. May/June 2008]

56. Find the Laplace transform of wave rectified sine wave defined as

$f(t) = a\ \text{sinpt}, 0 < t < \frac{\pi}{p}$

$= 0\ ,\ \frac{\pi}{p} < t < \frac{2\pi}{p}$ and $f(t+2\pi) = f(t)$ [F.E. May/June 2008]

57. Find: $L\{\text{cosat sinhat}\}$ [F.E. Nov/Dec 2006]

58. Find $L\{te^{-t}\text{sint}\}$ [F.E. Nov/Dec 2006]

59. Find $L\{e^{-3t}t^{-1/2}\}$ [F.E. Nov/Dec 2006]

60. Express into Heaviside function and hence find Laplace transform of

$f(t) = e^{t}\text{cost},\ 0 < t < \pi$

$= e^{t}\ \text{sint}\ , t > \pi$ [F.E. Nov/Dec 2006]

61. Find the L.T. of the triangular wave function of period 2c given by

$$f(t) = t, 0 < t < c$$
$$= 2c - t, c < t < 2c$$

62. Evaluate by Laplace transform: $\int_0^\infty e^{-t} \frac{\sin t}{t}\, dt$ [F.E. Nov/Dec 2006]
63. Find: $L\{t\, H(t-4) + t^2\, \delta(t-4)\}$ [F.E. Nov/Dec 2006]
64. Find: $L\{te^{-t}\sin 3t\}$ [F.E. May/June 2007]
65. Evaluate: $\int_0^\infty e^{-t}\, \frac{\sin^2 t}{t}\, dt$ [F.E. May/June 2007]
66. Find Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$ [F.E. May/June 2007]
67. Evaluate: $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}\, dt$ [F.E. May/June 2007]
68. Find: $L\{\, e^{-t} \int_0^t \sin(2t+3)\, dt\}$ [F.E. (Old)May/June 2008]
69. Find Laplace transform of:
    $f(t) = e^t, 0 < t < 2\pi$ and $f(t + 2\pi) = f(t)$ [F.E.(Old) (May/June 2008]
70. Find: $L[\sin(t-1)\, H(t-1)]$ [F.E.(Old) (May/June 2008]
71. Find: $L[t\, H(t-3) - t^3\, \delta(t-4)]$ [F.E.(Old) (May/June 2008]

Inverse Laplace Transform

1. $L^{-1}\left\{\frac{F(s)}{s}\right\} = \ldots\ldots$ [B.Tech. May/June 2013]
2. Solve ordinary differential equation by using Laplace $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 25y = 0$ with $y(0) = 0, y'(0) = -12$ [B.Tech. May/June 2013]
3. Using convolution theorem obtain inverse of $\frac{s^2}{(s^2+4)^2}$ [B.Tech. May/June 2013]
4. Find: $L^{-1}\left\{\text{Log}\frac{s+b}{s+a}\right\}$ [B.Tech. May/June 2013]
5. $L^{-1}\left\{\frac{e^{-\frac{\pi}{2}s}}{s^2+16}\right\} = \ldots\ldots$ [B.Tech.(Old) May/June 2013]
6. $L^{-1}\left\{\frac{1}{(3s+1)^2}\right\} = \ldots\ldots$ [B.Tech.(Old) May/June 2013]
7. Find: $L^{-1}\left\{\text{Log}\frac{1+s}{s}\right\}$ [B.Tech.(Old) May/June 2013]
8. Find the inverses of $\frac{1}{(s+1)(s^2+1)}$ by convolution method. [B.Tech.(Old) May/June 2013]
9. Find inverse Laplace transform of $\text{Log}\frac{s+a}{s+b}$ [B.Tech.Nov/Dec 2012]
10. $L^{-1}\left\{\frac{f(s)}{s}\right\} = \ldots\ldots$ [B.Tech.Nov/Dec 2012]
11. Find: $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$ [B.Tech.Nov/Dec 2012]
12. Find inverse Laplace transform of: $F(s) = \frac{s+2}{(s+1)(s+1)^3}$ [B.Tech.Nov/Dec 2012]
13. If $L^{-1}\{f(s)\} = f(t)$ then $L^{-1}\left\{\int_s^\infty f(s)ds\right\} = \ldots\ldots.$ [B.Tech.(Old)Nov/Dec 2012]

14. Solve: $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3t\,e^{-t}$ if $y(0) = 4, y'(0) = 2$ [B.Tech.(Old)Nov/Dec 2012]

15. Find: $L^{-1}\left\{\frac{1}{s^3(s+1)}\right\}$ [B.Tech.(Old)Nov/Dec 2012]

16. $L^{-1}\left\{\frac{e^{-2s}}{(s+4)^3}\right\} = \ldots\ldots$ [B.Tech. May/June 2012]

17. Find inverse Laplace transform of: $\text{Log}\sqrt{\frac{s-1}{s+1}}$ [B.Tech. May/June 2012]

18. Solve: $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = t^2\,e^{3t}$ if $y(0) = 2, y'(0) = 6$ [B.Tech. May/June 2012]

19. Find: $L^{-1}\{2\tanh^{-1}s\}$ [B.Tech. May/June 2012]

20. Solve by Laplace transform: $L\frac{di}{dt} + Ri = E,\ i(0) = 0$ [B.Tech.Nov/Dec 2009]

21. Find inverse Laplace transform of: $\frac{3s+1}{(s-1)(s^2+1)}$ [B.Tech.Nov/Dec 2009]

22. Find inverse Laplace transform of: $\frac{1}{(s+1)(s^2+1)}$ [B.Tech.Nov/Dec 2009]

23. Find: $L^{-1}\left[e^{-\frac{3\pi}{4}s}\left(\frac{s}{s^2+16}\right)\right]$ [B.Tech.Nov/Dec 2009]

24. Find: $L^{-1}\left[\frac{1}{2}\text{Log}\left(\frac{s^2+36}{s^2+16}\right)\right]$ [B.Tech.Nov/Dec 2009]

25. If $L^{-1}\{f(s)\} = f(t)$ and $L^{-1}\{g(s)\} = g(t)$ then
$L^{-1}\{f(s).g(s)\} = \ldots\ldots\ldots..$ [B.Tech.(Old)Nov/Dec 2009]

26. Find: $L^{-1}\left\{\tan^{-1}\frac{a}{s}\right\}$ [B.Tech.(Old)Nov/Dec 2009]

27. Find: $L^{-1}\left\{\frac{1}{s\ (s+1)^3}\right\}$ [B.Tech.(Old)Nov/Dec 2009]

28. Solve the differential equation: $y'' + 4y' + 3y = e^{-t}$, $y(0) = y'(0) = 1$

29. Find: $L^{-1}\left\{\text{Log}\left(1 - \frac{a^2}{s^2}\right)\right\}$ [B.Tech. May/June 2010]

30. If $L^{-1}\{f_1(s)\} = f_1(t)$ and $L^{-1}\{f_2(s)\} = f_2(t)$ then
$L^{-1}\{f(_1(s)).f_2(s)\} = \ldots\ldots\ldots..$ [B.Tech. May/June 2011]

31. Find: $L^{-1}\left\{\frac{3(s^2-1)^2}{2s^5}\right\}$ [B.Tech. May/June 2011]

32. Solve the equation: $y'' - 3y' + 2y = 4t + e^{3t}$ when $y(0) = 4, y'(0) = 1$

33. Find: $L^{-1}\left[\frac{e^{-s}}{(s+1)^2}\right]$ [B.Tech. May/June 2011]

34. Find: $L^{-1}\left[\frac{s^3}{s^4-a^4}\right]$ [B.Tech.Nov/Dec 2010]

35. Find: $L^{-1}\left[\frac{s}{(s+1)^3}\right]$ [B.Tech.Nov/Dec 2010]

36. Find: $L^{-1}\left[\frac{1}{9s^2+6s+1}\right]$ [B.Tech.Nov/Dec 2011]

37. Find inverse Lapalace transform of : $\cot^{-1}\left(\frac{s}{a}\right)$ [F.E.May/June 2013]

38. Use convolution theorem to find : $L^{-1}\left(\frac{1}{s^4-2s^3}\right)$ [F.E.May/June 2013]

39. Solve by using Laplace transform: $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 3\cos3t - 11\sin3t$

if $y(0) = 0, y'(0) = 6$ [F.E.May/June 2013]

40. Use convolution theorem to find : $L^{-1}\left(\frac{s^2-a^2}{(s^2+a^2)^2}\right)$ [F.E.May/June 2008]

41. Solve by using Laplace transform: $(D^2 + 2D + 6)y = e^{-t}\sin t$

42. where $y(0) = 0, y'(0) = 1$ [F.E.May/June 2008]

43. Find: $L^{-1}\left(\frac{1}{s^2+2s+2}\right)$ [F.E.Nov/Dec 2006]

44. In an electrical circuit with e.m.f. $E(t)$ resistance R and indunctance L the current i builds up at the rate given by $L\frac{di}{dt} + Ri = E(t)$. If the switch is connected at $t = 0$ and disconnected at $t = a$, find the current i at any instant. [F.E.Nov/Dec 2006]

45. Find: $L^{-1}\left[\frac{1}{(s^2+4)^2}\right]$ [F.E.Nov/Dec 2006]

46. Find: $L^{-1}\left[\frac{1}{s^2(s+1)^2}\right]$ [F.E.May/June 2007]

47. Find inverse Lapalace transform of : $\frac{e^{-\frac{4\pi s}{6}}s}{s^2+25}$ [F.E.May/June 2007]

48. Find inverse Lapalace transform of : $\frac{e^{-\pi s/2}+e^{-3\pi s/2}}{s^2+1}$ [F.E.May/June 2007]

49. Using convolution theorem obtain inverse of $\frac{s}{(s^2+a^2)^2}$ [F.E.(Old)May/June 2008]

50. Find: $L^{-1}\left[\frac{e^{-as}}{(s+b)^{5/2}}\right]$ [F.E.(Old)May/June 2008]

Unit-V Fourier Transform

1. If Fourier sine transform $F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(t)\sin st\, dt$ then F. inverse sine transform is given by……….. [B.Tech.May/June 2013]
2. Find the Fourier integral representation of the function of the function $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ Hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\, d\lambda$. [B.Tech.May/June 2013]
3. Using Fourier integral representation, show that: $\int_0^\infty \frac{\lambda\sin\lambda x}{\lambda^2+m^2} d\lambda = \frac{\pi}{2} e^{-mx}$, $m > 0$, $x > 0$ [B.Tech.May/June 2013]
4. Find Fourier transform of $\frac{1}{1+x^2}$ [B.Tech.May/June 2013]
5. Show that: $\int_0^\infty \frac{\sin\pi s \sin sx}{1-s^2} ds = \frac{\pi}{2}\sin x , x < \pi$
   $= 0 ,\ x > \pi$ [B.Tech(Old)May/June 2013]
6. Find Fourier cosine transform of $\frac{1}{1+x^2}$ [B.Tech(Old)May/June 2013]
7. Find Fourier transform of $f(x) = 1$,$0 < x < a$
   $= 0$, otherwise [B.Tech.Nov/Dec 2012]
8. Find Fourier cosine transform of $f(x) = 1$, $0 < x < \frac{\pi}{2}$ [B.Tech.Nov/Dec 2012]

9. Find the Fourier transform of

$$f(x) = 1 - x^2 \,, |x| \le 1$$
$$= 0, |x| \ge 1$$

Hence evaluate: $\int_0^\infty \frac{\sin x - x\cos x}{x^3} \cos\frac{x}{2}\, dx$ [B.Tech.Nov/Dec 2012]

10. Find Fourier cosine and sine transform of :

$f(x) = \sin ax \,, 0 < x < \pi$ [B.Tech.Nov/Dec 2012]

11. Write formula for Fourier cosine transform…… [B.Tech(Old)Nov/Dec 2012]
12. Find cosine transform of e^{-2x} [B.Tech(Old)Nov/Dec 2012]

13. Using Fourier integral representation show that:

$$\int_0^\infty \frac{\cos\frac{\pi}{2}\lambda \cos\lambda x}{1-\lambda^2} d\lambda = \frac{\pi}{2}\cos x \,, |x| \le \frac{\pi}{2}$$
$$= 0 \,, |x| \ge \frac{\pi}{2}$$

[B.Tech(Old)Nov/Dec 2012]

14. Find the Fourier transform of $f(x) = e^{-x^2}$ [B.Tech(Old)Nov/Dec 2012]
15. Fourier cosine transform $f_c(\lambda) =$……….. [B.Tech.May/June 2012]
16. The Fourier transform of

$f(x) = \begin{cases} e^{-ax}, & 0 < x < \infty \\ 0, & x < 0 \end{cases}$ is ……. [B.Tech.May/June 2012]

17. Find f(x) if its Fourier sine transform is $\frac{\lambda}{\lambda^2+1}$ [B.Tech.May/June 2012]
18. Find the Fourier sine transform of $e^{-|x|}$ hence deduce that

$\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$ [B.Tech.May/June 2012]

19. The complex form of Fourier integral is given by the formula…….
[B.Tech.Nov/Dec 2009]
20. The inverse Fourier sine transform is stated as….. [B.Tech.Nov/Dec 2009]
21. $F[f(x)\, g(x)] =$……….. [B.Tech.Nov/Dec 2009]
22. Find the Fourier transform of the slit function defined as

$$f(t) = 1 \,, |t| \le a$$
$$= 0, |t| > a$$

[B.Tech.Nov/Dec 2009]

23. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ [B.Tech.Nov/Dec 2009]
24. If $F[f(x)] = F(s)$ then $F[f(x-a)] =$…… [B.Tech.May/June 2010]
25. If $F[f(x)] = F(s)$ then $F_s[f(x)\sin ax] =$…….. [B.Tech.May/June 2010]
26. The inverse Fourier cosine transform is stated as….. [B.Tech.May/June 2010]
27. Find the Fourier transform of :

$f(x) = x^2 \,, |x| \le a$

$= 0, |x| > a$ [B.Tech.May/June 2010]

28. Find Fourier sine integral of $e^{-\beta x}$ [B.Tech.May/June 2010]
29. If $F[f(x)] = F(s)$ then $F_s[f(x)\cos ax] =$…….. [B.Tech.May/June 2011]
30. If $F[f(x)] = F(s)$ then $F_c[f(x)\sin ax] =$…….. [B.Tech.May/June 2011]
31. Find the Fourier transform of :

    $f(x) = 1 , |x| \leq a$

    $= 0, |x| > a$ [B.Tech.May/June 2011]
32. Find Fourier cosine transform of $e^{-|x|}$ [B.Tech.May/June 2011]
33. Fourier cosine integral of f(x) is given by…….. [B.Tech.Nov/Dec 2010]
34. If $F[f(x)] = F(s)$ then $F\,[f(x)\cos ax] =$…….. [B.Tech.Nov/Dec 2010]
35. Find the Fourier transform of :

    $f(x) = x , |x| \leq a$

    $= 0, |x| > a$ [B.Tech.Nov/Dec 2010]

36. Find Fourier sine transform of $e^{-|x|}$ [B.Tech.Nov/Dec 2010]
37. If $F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$, then $f(x) =$ ……..

    [B.Tech.Nov/Dec 2011]
38. If $F_s(s)$ and $F_c(s)$ are Fourier sine and cosine transforms of f(x) then

    $F_s[f(x)\sin ax] =$…….. [B.Tech.Nov/Dec 2011]
39. Find the Fourier transform of :

    $f(x) = 1 , |x| < 1$

    $= 0, |x| > 1$

    Hence evaluate $\int_0^{\infty} \frac{\sin x}{x}\, dx$ [B.Tech.Nov/Dec 2011]

40. Find the Fourier transform of

    $f(x) = \sin x, 0 < x < \pi$

    $= 0, x > \pi, x < 0$

    Hence deduce that $\int_0^{\infty} \frac{\cos\left(\frac{\pi w}{2}\right)}{1-w^2}\, dw = \frac{\pi}{2}$ [F.E. 2013]

41. Find the Fourier sine transform of :

    $F(x) = x, a \leq x \leq b$

    $= 0,$ otherwise [F.E. 2013]
42. Express :$f(x) = 1 , \; 0 < x < \pi$

    $= 0 \;, x > \pi$

    as a Fourier integral and hence evaluate $\int_0^{\infty} \frac{(1-\cos \pi\lambda)\sin(\lambda x)}{\lambda}\, d\lambda$ [F.E. 2008]
43. Using Fourier transform show that

    $e^{-x}\cos x = \frac{2}{\pi} \int_0^{\infty} \frac{(u^2+2)\cos xu}{u^4+4}\, du$ for $x > 0$ [F.E. 2008]

44. Find the Fourier cosine transform of : $2e^{-3x} + 4e^{-2x}$ [F.E. 2008]

45. Find f(x) if $F_s(s) = \frac{e^{-as}}{s}$ [F.E. 2008]

46. Find the Fourier cosine transform of e^{-x^2} [F.E. 2006]

47. Find Fourier sine transform of :

$f(x) = \cos x, 0 \le x \le a$

$= 0, \ x > a$ [F.E. 2006]

48. Find Fourier cosine integral representation for the function;

$f(x) = x^2 , 0 < x < a$

$= 0, x > a$ [F.E. 2007]

49. Using inverse sine transform, find f(x) if $f_s(\lambda) = \frac{e^{-a\lambda}}{\lambda}$ [F.E. 2007]

50. Using Fourier integral representation show that

$\int_0^\infty \frac{\cos\frac{\pi}{2}\lambda \cos\lambda x}{1-\lambda^2} d\lambda = \frac{\pi}{2} \cos x , |x| \le \frac{\pi}{2}$

$= 0 , |x| > \frac{\pi}{2}$ [F.E. 2008]

51. Find Fourier sine integral of $f(x) = e^{-\beta x}$ ($\beta > 0$)

Hence show that $\int_0^\infty \frac{\lambda \sin\lambda x}{\beta^2+\lambda^2} d\lambda = \frac{\pi}{2} e^{-\beta x}$ [F.E. 2008]

52. Using Fourier integral representation show that

$\int_0^\infty \frac{\cos\lambda x}{1+\pi^2} d\lambda = \frac{\pi}{2} e^{-x}$, $(x > 0)$ [F.E. 2008]

Unit-VI Probability

1. The Binomial Distribution whose mean is 5 and variance $=\frac{10}{3}$ is ………..
2. Out 800 families with 5 children each , how many families would you expected to have :

   i) 03 boys and 02 girls ii) 02 boys and 03 girls iii) at the most 2 girls
3. In a sample of 1000 cases the mean of a certain test is 14 and S.D. = 2.5.Assuming the

   distribution to be normal find :

   i) How many students score between 12 and 15?

   ii) How many score above 18

   iii) How many score below 18, Given that SNV for 0.8 is 0.2881, SNV for 0.4 is 0.1554

   SNV for 1.6 is 0.4452, SNV for 2.4 is 0.4318
4. For Poisson distribution mean =….. …and variance =…………,S.D.= ………
5. Six coins are tossed 320 times .Using Poisson distribution find approximately the probability of getting 6 heads at most 2 times.
6. For Poisson distribution P(r) = ………..

7. If the probability of bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get bad reaction.
8. The probability that a man aged 60 will live to be 70 is 0.065 what is the probability that out of 10 men now at 60 at least 7 would live to be 70.
9. The life time of certain type of battery has mean life of 400 hours and a standard deviation of 50 hours. Assuming the distribution of life time to be normal. Find the percentage of batteries which have life more than 350 hours (The area under normal curve from z=0 t0 z=1 is 0.3413)
10. A manufacturer knows from experience that the resistance of resistors he produces is normal with mean $\mu = 100$ ohms and standard deviation $= 2$ ohms . What is the percentage of resistors will have resistance between 98 ohms and 102 ohms .
11. A manufacturer knows that the condensers he makes contain on an average 1% defectives. he packs them in a box 0f 100. What is the probability that a box picked at random will contain 4 or more faulty condensers?
12. In a male population of 1000, the mean height is 68.16 inches and S.D. 3.2 inches .How many men may be more than 6 feet.
    [$\Phi(1.15) = 0.8749$, $\Phi(1.2) = 0.8849$, $\Phi(1.25)=0.8944$] where the argument is the standard normal variable.]
13. If 10% of bolts produced by a machine are defective ,determine the probability that out of 10 bolts ,chosen at random
    i)one ii) none iii) at most two bolts will be defective .

14. A die is thrown 8 times and it is required to find the probability that 3 will show:

i) Exactly 2 times ii) At least seven times iii) Atleast once

15. If on an average one ship in ten every ten is wrecked, find the probability that out of 5 ships expected to arrive, at least four will arrive safely.

16. Articles of which 2 percent are defective are packed in boxes each containing 200 articles, using Poisson distribution find the chance that boxes will contain:

(i) No defective (ii) Two or more defective

17. If the probability of the defective fuse is 0.05, the variance for Poisson distribution of defective fuses in total of 40 is ………….

18. If the probability of the defective bolt is 0.0001 for the distribution of defective bolts in total of 1000 then mean=……….

19. Two percent of screws produced in a certain factory turn out to be defective.Find the probability that in a sample of 10 screws chosen at random, exactly two will be defective.

20. In a test of 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 Hrs and Std. deviation of 60 Hrs. Estimate the no. of bulbs likely to burn for, more than 2150 Hrs.

[SNV of $z = 1.83$ is 0.4664]

21. The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped the probability that at least two will strike the target is ……….

22. Assuming that the diameter 1000 brass plugs taken consecutively from a machine form a normal distribution with mean 0.7515 inch and S.D. is 0.002 inch. How many plugs are likely to be rejected if approved diameters are 0.7520 ± 0.004 ?

23. Find the binomial distribution whose mean is 5 and S.D. is 2.

24. Find the probability that at most 5 defective fuses will be found in a box of 200 fuses of 20% of such fuses are defective.

25. six coins are tossed 320 times. Using Poisson distribution find approximately the probability of getting 6 heads at most two times.

26. For Binomial distribution mean=……, variance=………. and S.D.=……

27. For Binomial distribution P(r) = …….

28. In a certain examination test, 2000 students appeared in a subject of maths. Avearge marks obtained are 50% with S.D. 2% . How many students are expected to obtain more than 60% of marks. Asuuming that the marks are distributed normally. Area under curve from $z = 0$ to $z = 2$

is 0.4772.

29. Out 800 families with 4 children each , how many families would you expected to have at least a boy.

30. For a binomial distribution the mean is 2 and S.D. is 1 find p and q.

31. If a dice is thrown what is the probability that outcome will be either number 2 or 3.

32. From a box containing 100 transistors 20 of which are defective 10 are selected at random. Find the probability that :

(i) all will be defective (ii) At least on defective using binomial distribution.

33. If a chance that out of ten telephone lines one of the line busy at an instant is 0.2 :

(i) what is the chance that five of the lines are busy?

(ii) what is the probability that all of the lines are busy?

34. 5000 candidates appeared in a certain paper carrying maximum of 100 marks was found that marks were normally distributed with mean 39.5 and S.D. 12.5. Determine approximately the number of candidates who secured first class for which a minimum of 60 marks is necessary.

35. In the normal distribution of 700 items, mean is 95 and S.d. is 15. How many items lie between 80 and 90? The area under the normal curve for -1 is 0.3413 and -0.3333 is 0.1280.

35.Five thousand candidates appeared in a certain paper carrying maximum of 100 marks was found that marks were normally distributed with mean 39.5 and S.D. 12.5. Determine approximately the number of candidates who scored 60 or above marks.

36. Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six.

36. A firm produces article of which 0.1 percent are defective. It packs them in cases each containing 500 articles, if a saler purchases 100 such cases, how many cases can be expected to be free from defective , how many can be expected to have one defective.

37. The probability that an evening college student will graduate is 0.4. determine the probability that out of 5 students :

(i) none (ii) one (iii) At least one will graduade.

38. If the mean life time and S.D. of battery cells is 24 hrs and 3 hrs, what percentage of batteries will have life:

(i) Between 10 and 14 hrs (ii) more than 15 hrs (iii) less than 6 hrs ?

39. Out 1000 families with 3 children each , how many families would you expected to have :

(i) 02 boys and 01 girls (ii) 2 girl and 1 boy?

40. The probability that a man aged 40 years will die within next year is 0.001. What is the probability that out 100 such persons at least 99 will survive till next year ?

41. The daily sales of certain item are normally distributed with mean Rs. 8000 and variance Rs.10,000

(i) what is the probability that on a certain day the sales be less than Rs.8210

(ii) what percentage of the days will be sales between Rs.8100 and Rs.8210 ?

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