UNIT-I Matrices

1. Define Rank of matrix
2. Find the rank of the matrix: $\begin{bmatrix} 1 & 2 \\ 7 & 14 \end{bmatrix}$ [Ans: 1]
3. Find the rank of the matrix: $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$ [Ans: 3]
4. Determine the rank of the matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ [Ans: 2]
5. Find the rank of the matrix: $\begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix}$ [Ans: 1]
6. Find the rank of the matrix: $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$ [Ans: 2]
7. Find the rank of the matrix: $\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$ [Ans: 3]
8. Find the rank of the matrix: $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ [Ans: 3]
9. Determine the rank of the matrix: $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ [Ans: 2]
10. Determine the rank of the matrix: $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ [Ans: 2]
11. Find the rank of the matrix: $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$ [Ans: 2]
12. Find the rank of $A + B, AB$ and BA if $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 10 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ [Ans: 3,1,1]

1. Define normal form of matrix.

2. Find the rank of the matrix by reducing to its normal form: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ [Ans: 3]

3. Reduce the following matrix into its normal form and hence find its rank

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ [Ans: 3]

4. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ Find two non singular matrices P and Q such that PAQ $= I$

Hence find A^{-1}

Ans: $P = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ -2 & 3 & -3 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

5. Find non singular matrices P and Q such that PAQ is a normal form where

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Ans: $P = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -\frac{3}{14} & \frac{1}{28} & \frac{9}{28} \end{bmatrix}$, $Q = \begin{bmatrix} 1 & -1 & 4 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

6. Find P and Q such that the normal of $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is PAQ

Ans: $P = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

7. Find non singular matrices P and Q such that PAQ is a normal form where

$$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

Ans: $P = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & -3 & -9 & 7 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

8. Solve the following equations

$x - y + 2z = 3 , x + 2y + 3z = 5 , 3x - 4y - 5z = -13$ [Ans: $x = -1, y = 0, z = 2$]

9. Discuss the consistency and solve

$2x + 3y + 4z = 11 , x + 5y + 7z = 15 , 3x + 11y + 13z = 25$

[Ans: $x = 2, y = -3, z = 4$]

10. Solve: $3x + 3y + 2z = 1 , x + 2y = 4 , 10y + 3z = -2 , 2x - 3y - z = 5$

[Ans: $x = 2, y = 1, z = -4$]

11. Test the consistency and solve

$5x + 3y + 7z = 4 , 3x + 26y + 2z = 9 , 7x + 2y + 10z = 5$

[Ans: $x = \frac{7}{11}, y = \frac{3}{11}, z = 0$]

12. Test the consistency and solve

$x_1 + x_2 + x_3 + x_4 = 0 ,\qquad x_1 + x_2 + x_3 - x_4 = 4$

$x_1 + x_2 - x_3 + x_4 = -4 ,\qquad x_1 - x_2 + x_3 + x_4 = 2$

[Ans: $x_1 = 1, x_2 = -1, x_3 = 2 , x_4 = -2$]

13. Solve the following equations

$2x_1 + x_2 + 2x_3 + x_4 = 6 ,\qquad 6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$

$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1 , 2x_1 + 2x_2 - x_3 + x_4 = 10$

[Ans: $x_1 = 2, x_2 = 1, x_3 = -1 , x_4 = 3$]

14. Solve: $x_1 + x_2 - x_3 = 0 , 2x_1 - x_2 + x_3 = 3 , 4x_1 + 2x_2 - 2x_3 = 2$

[Ans: $x_1 = 1, x_2 = k - 1, x_3 = k$]

15. Solve the following equations

$x_1 + 2x_2 - x_3 = 1 , 3x_1 - 2x_2 + 2x_3 = 2, 7x_1 - 2x_2 + 3x_3 = 5$

[Ans: $x_1 = -\frac{k}{4} + \frac{3}{4} , x_2 = \frac{5k+1}{8}, x_3 = k$]

16. Test for consistency and solve

$5x + 3y + 7z = 4 , 3x + 26y + 2z = 9 , 7x + 2y + 10z = 5$

[Ans: $x = -\frac{16}{11}k + \frac{7}{11}, y = \frac{3}{11} + \frac{k}{11}, z = k$]

17. Determine the values of a and b for which the system

$x + 2y + 3z = 6 , x + 3y + 5z = 9 , 2x + 5y + az = b$

has (i) no solution (ii) unique solution (iii) infinite number of solutions

18. Find for what values of k the set of equations

$2x - 3y + 6z - 5t = 3 , y - 4z + t = 1 , 4x - 5y + 8z - 9t = k$

has (i) no solution (ii) infinite number of solutions

19. Solve: $x + 2y + 3z = 0 , 3x + 4y + 4z = 0 , 7x + 10y + 12z = 0$

Ans: $x = y = z = 0$

20. Solve : $x_1 + 3x_2 + 2x_3 = 0 ,\ 2x_1 - x_2 + 3x_3 = 0$

$3x_1 - 5x_2 + 4x_3 = 0,\qquad x_1 + 17x_2 + 4x_3 = 0$

[Ans: $x_1 = 11k, x_2 = k, x_3 = -7k$]

21. Solve: $x + y - 3z + 2w = 0 , 2x - y + 2z - 3w = 0$

$3x - 2y + z - 4w = 0 , -4x + y - 3z + w = 0$

Ans: $x = y = z = w = 0$

22. Solve the system of homogeneous equations

$4x + 2y + z + 3w = 0, 6x + 3y + 4z + 7w = 0 , 2x + y + w = 0$

Ans: $x = k_1 , y = -2k_1 - k_2 , z = -k_1 , w = k_2$

23. Determine the values of λ such that the system of equations $2x + y + 2z = 0 , x + y + 3z = 0, 4x + 3y + \lambda z = 0$ have non zero solution

    Ans: $\lambda = 8$

24. Find the values of k such that the system of equations $x + ky + 3z = 0, 4x + 3y + kz = 0,2x + y + 2z = 0$

    Ans: $k = 0 , \frac{9}{2}$

25. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ find the Eigen values of $3A^3 + 5A^2 - 6A + 2I$

    Ans: $\lambda = 1,3,-2 \;:\; 4,\; 110 ,10$

26. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

    Ans: $\lambda = 1,2,3 : \quad X = \begin{bmatrix} k \\ -k \\ 0 \end{bmatrix}, \begin{bmatrix} 2k \\ -k \\ -2k \end{bmatrix}, \begin{bmatrix} k \\ -k \\ -2k \end{bmatrix}$

27. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

    Ans: $\lambda = -2,3,6 : \quad X = \begin{bmatrix} -k \\ 0 \\ k \end{bmatrix}, \begin{bmatrix} k \\ -k \\ k \end{bmatrix}, \begin{bmatrix} k \\ 2k \\ k \end{bmatrix}$

28. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

    Ans: $\lambda = 2,3,5: \quad X = \begin{bmatrix} k \\ -k \\ 0 \end{bmatrix}, \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3k \\ 2k \\ k \end{bmatrix}$

29. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

    Ans: $\lambda = 1,2,2 : X = \begin{bmatrix} k \\ k \\ -k \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

30. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

    Ans: $\lambda = 5, -3, -3 : \quad X = \begin{bmatrix} k \\ 2k \\ -k \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

31. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

Ans: $\lambda = 0,1,1$: $X = \begin{bmatrix} 2k \\ -k \\ 2k \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$

32. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

Ans: $\lambda = 1,1,1$: $X = \begin{bmatrix} k \\ k \\ k \end{bmatrix}$

33. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Ans: $\lambda = 3,1,1$: $X = \begin{bmatrix} k \\ k \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

34. Define the following terms
   (a) Unique solution
   (b) Trivial solution
   (c) Consistency of equations
   (d) Characteristic equation
   (e) Eigen Values OR Characteristic roots
   (f) Eigen Vectors OR Characteristic vectors

35. State Cayley-Hamilton theorem.

36. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ hence find A^{-1}

Ans: $\lambda^3 - \lambda^2 - 4\lambda + 4 = 0$, $A^{-1} = \frac{1}{4}\begin{bmatrix} 4 & 4 & -4 \\ -2 & -1 & 3 \\ -2 & 1 & 1 \end{bmatrix}$

37. Find the characteristic equation of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ hence find A^{-1}

Ans: $\lambda^3 - 6\lambda^2 + 6\lambda - 11 = 0$, $A^{-1} = \frac{1}{11}\begin{bmatrix} 5 & -1 & -7 \\ 1 & 3 & 10 \\ -1 & 1 & -2 \end{bmatrix}$

38. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verified that it is satisfied by A and hence obtain A^{-1}

Express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ in linear polynomial in A

Ans: $\lambda^3 - 6\lambda^2 + 9\lambda - 4I = 0$, $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$: $5A - I$

39. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and verify

Cayley-Hamilton theorem and hence evaluate

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

Ans: $\lambda^3 - 5\lambda^2 + 7\lambda - 3I = 0$, $\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$

40. Define linear dependence and linear independence of vectors.

41. Is the system of vectors $X_1 = (2,2,1)^T$, $X_2 = (1,3,1)^T$, $X_3 = (1,2,2)^T$ linearly dependent.

42. Examine the vectors $(1 , 2, 4), (2, -1,3), (0,1,2), (-3,72)$ for linear dependence and find the relation if it exists.

Ans: $9X_1 - 12X_2 + 5X_3 - 5X_4 = 0$

43. Examine the vectors $[1,0,2,1]$, $[3,1,2,1], [4,6,2,-4], [-6,0,-3,-4]$ for linear dependence and find the relation if it exists.

Ans: $2X_1 - 6X_2 + X_3 - 2X_4 = 0$

UNIT-II Infinite Series

1. Test the convergence of the series : $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \cdots \infty$ [B.tech.Old Nov/Dec 2012]
2. Discuss the convergence of the series: $\sum_{n=1}^{\infty} \left(\frac{n+1}{3n}\right)^n$ [B.tech.Old Nov/Dec 2012]
3. Test the convergence of the series :$\sum_{n=2}^{\infty} \frac{1}{n\ logn}$ [B.tech.Old Nov/Dec 2012]
4. Examine the convergence of the sequence: $a_n = \frac{1}{2n}$ [B.tech.Nov/Dec 2012]
5. Test the convergence of the series :$\sum_{n=1}^{\infty} \frac{2n^2+3n}{5+n^5}$ [B.tech.Nov/Dec 2012]
6. If $u_n = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ then u_n is ……….. nature [B.tech.Nov/Dec 2012]
7. Test for convergence for series: $\frac{2}{1} + \frac{2.5.8}{1.5.9} + \frac{2.5.8.11}{1.5.9.13} + \cdots \infty$ [B.tech.Nov/Dec 2012]
8. Test the convergence for series whose nth term is : $\frac{(n+1)}{n^2}\ x^n$ [B.tech.Nov/Dec 2012]
9. Discuss the convergence of the series: $\sum_{n=1}^{\infty} \frac{1}{(logn)^{\ n}}$ [B.tech.Nov/Dec 2012]
10. Test the convergence of the series: $\sum_{n=1}^{\infty} (\ 1\ +\ \frac{1}{n}\ )^{\ -n^{\ 2}}$ [B.tech.Nov/Dec 2008]
11. Test the convergence for series whose nth term is : $\frac{n^3}{(n-1)!}$ [B.tech.Nov/Dec 2010]
12. Test the convergence of the series :$\frac{2}{1} + \frac{3}{8} + \frac{4}{27} + \frac{5}{64} + \cdots \infty$ [B.tech.Nov/Dec 2010]
13. Test the convergence of the series :$\frac{2}{7} + \frac{2.5}{7.10} + \frac{2.5.8}{7.10.13} + \cdots$ [B.tech.Nov/Dec 2010]
14. Discuss the convergence of the series : $\sum_{n=1}^{\infty} \frac{n!}{n^{\ n}}$ [B.tech.Nov/Dec 2008]
15. State with reasons the values of x for which the series

$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}$.... Converges [B.Tech May/June 2009]

16. Discuss the convergence of the series : $\sum_{n=1}^{\infty}\frac{n^{2}}{3^{n}}$ [B.Tech May/June 2009]
17. Test the convergence of the series : $\sum_{n=1}^{\infty}\frac{n^{4}}{n^{6}+1}$ [B.Tech May/June 2009]
18. Test the convergence of the series: $\sum_{n=0}^{\infty}(-1)^{n}$ [B.tech.May/June 2010]
19. Test for convergence for series: $\frac{1}{1.2.3}+\frac{3}{2.3.5}+\frac{5}{3.4.5}+\cdots\infty$ [B.tech.May/June 2010]
20. Using integral test discuss the convergence of the series: $\sum_{n=1}^{\infty}\frac{1}{n^{2}}$
21. Test the convergence of the series : $\sum_{n=1}^{\infty}\frac{2^{n}-1}{2^{n}}$
22. Discuss the convergence of the series : $\sum_{n=1}^{\infty}\frac{1}{n^{n}}$
23. Discuss the convergence of the series : $\sum_{n=1}^{\infty}\frac{n^{2}+2^{n}}{2^{n}\,n^{2}}$ [B.tech.May/June 2010]
24. Test the convergence of the series: 1 + 4 + 7 + 10 + …………∞
25. The ratio test for the positive term series $\sum_{n=1}^{\infty}a_n$ states that…… [F.E.2013]
26. Discuss the convergence of the series : $\sum_{n=1}^{\infty}\frac{n^{2}}{2^{n}}$ [F.E.2013]

UNIT-III Successive Differentiation

1. Evaluate: $\lim_{x\to 0}\left[\frac{1}{Logx}-\frac{x}{x-1}\right]$ [B.Tech.(Old) Nov/Dec 2012]
2. Obtain expansion of e^x in powers of $(x-2)$. [B.Tech.(Old) Nov/Dec 2012]
3. Using Maclaurin's series expand $sinhx$ [B.Tech. Nov/Dec 2012]
4. Evaluate: $\lim_{x\to 0}\left(\frac{1}{x^2}-cot^{2}x\right)$ [B.Tech. Nov/Dec 2012]
5. Prove that: $e^{xsinx}=1+x^2+\frac{x^4}{3}+$……. [B.Tech. Nov/Dec 2012]
6. Show that: $sinh^{-1}x=x-\frac{x^3}{6}+\frac{3}{40}x^5-$…. [B.Tech. Nov/Dec 2008]
7. Evaluate: $\lim_{x\to 0}\frac{1-x^x}{x.logx}$ [B.Tech. Nov/Dec 2008]
8. Evaluate: $\lim_{x\to\infty} e^{\frac{sinh^{-1}x}{cosh^{-1}x}}$ [B.Tech. Nov/Dec 2010]
9. Prove that: $Log(x\,cotx)=-\frac{x^2}{3}-\frac{7}{90}x^4-\cdots$ [B.Tech. Nov/Dec 2010]
10. Prove that: $Log(1+e^x)=Log2+\frac{x}{2}+\frac{x^2}{8}-\frac{x^4}{192}+$……. [B.Tech. May/June 2007]
11. Evaluate: $\lim_{x\to\frac{\pi}{2}}(cosecx)^{tan^{2}x}$ [B.Tech. May/June 2007]
12. Expand $tan^{-1}x$ in powers of $(x-1)$ [B.Tech. May/June 2007]
13. Evaluate: $\lim_{y\to 1}\frac{y^y-y}{y-1-logy}$ [B.Tech. May/June 2009]
14. Prove that: $e^{\theta}=1+tan\theta+\frac{tan^{2}\theta}{2!}-\frac{tan^{3}\theta}{3!}+$…….. [B.Tech. May/June 2009]
15. Expand $sin^{-1}x$ in ascending powers of x [B.Tech. May/June 2009]

16. Prove that: $cosx\ coshx = 1 - \frac{2^2x^4}{4!} + \frac{2^4x^8}{8!} + \ldots\ldots$ [B.Tech. May/June 2009]

17. Evaluate: $\lim_{x\to a}\left(2 - \frac{x}{a}\right)\tan\left(\frac{\pi x}{2a}\right)$ [B.Tech.Nov/Dec 2009]

18. Expand e^x in powers of $(x-1)$ [B.Tech. May/June 2010]

19. Find the value of a and b such that : $\lim_{x\to 0}(x^{-3}sin3x + ax^{-2} + b) = 0$

20. Obtain expansion of $cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)$ in ascending powers of x [B.Tech. May/June 2010]

21. Expand $(1+x)^{1/x}$ upto the terms containing x^2 [B.Tech. May/June 2008]

22. Evaluate: $\lim_{x\to 1}(1-x^2)\ \frac{1}{Log(1-x)}$ [B.Tech. May/June 2008]

23. Evaluate: $\lim_{x\to 0}\frac{e^{xsinx} - \cosh(x\sqrt{2})}{x^4}$ [B.Tech. May/June 2008]

24. Find $\lim_{x\to a}\log\left(2 - \frac{x}{a}\right)\cot(x-a)$ [B.Tech.Nov/Dec 2009]

25. Evaluate: $\lim_{x\to 0}\frac{xe^x - Log(1+x)}{x^2}$ [B.Tech. May/June 2012]

26. Expand $log_e x$ in powers of $(x-1)$ [B.Tech. May/June 2012]

27. Evaluate: $\lim_{x\to 0}\frac{Logx}{cotx}$ [B.Tech. May/June 2012]

28. Using Maclaurin's series expand $sinx$ in ascending powers of x

29. Expand e^{sinx} upto the terms containing x^4 [B.Tech. May/June 2012]

30. Evaluate: $\lim_{x\to 0}\frac{1-cosx}{x^2}$ [F.E. May/June 2013]

31. Prove that: $Log(1+sinx) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \ldots\ldots$ [F.E. May/June 2013]

32. Evaluate: $\lim_{x\to 0}\frac{coshx - cosx}{x\ sinx}$ [F.E. May/June 2013]

33. Prove that:

$Logx = Log2 + \frac{x-2}{2} - \frac{(x-2)^2}{8} + \frac{(x-2)^3}{24} - \ldots..$ [F.E. Nov/Dec 2009]

34. Prove that: $sec^2x = 1 + x^2 + \frac{2}{3}x^4 + \ldots..$ [F.E. Nov/Dec 2009]

35. Prove that: $\lim_{x\to 0}\left[cotx - \frac{1}{x}\right] = 0$ [F.E. Nov/Dec 2009]

36. Prove that: $\lim_{x\to 0}\left(\frac{sinx}{x}\right)^{1/x} = 1$ [F.E. Nov/Dec 2009]

37. Prove that: $Logx = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 \ldots\ldots..$[F.E. May/June 2012]

38. Evaluate: $\lim_{x\to 0}(1+tanx)^{cotx}$ [F.E. May/June 2012]

39. Use Taylor's theorem to find $\sqrt{25.15}$ [F.E. Nov/Dec 2008]

40. Find: $\lim_{x\to 0}\frac{(1+x)^{1/x} - e + \frac{e^x}{2}}{x^2}$ [F.E. Nov/Dec 2008]

41. Prove that: $Log(1+e^x) = Log2 + \frac{x}{2} + \frac{x^2}{2}\ldots\ldots\ldots$ [F.E. Nov/Dec 2008]

42. Prove that: $sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\right]$ [F.E. May/June 2008]

43. Evaluate: $\lim_{x\to 0} \frac{sinx\ sin^{-1}x}{x^2}$ [F.E. May/June 2008]

44. Evaluate: $\lim_{x\to 0} (cos^2 x)^{1/x^2}$ [F.E. May/June 2008]

45. Evaluate: $\lim_{x\to 1} (1-x)\tan\left(\frac{\pi x}{2}\right)$ [F.E. Nov/Dec 2007]

46. Expand $cos^2 x$ in ascending powers of x [F.E. Nov/Dec 2007]

47. Prove that: $Log(secx) = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{2}{90}x^6 + \cdots$ [F.E. Nov/Dec 2007]

48. Evaluate: $\lim_{x\to 0} xLogx$ [B.Tech. May/June 2014]

49. Evaluate: $\lim_{x\to 1} (1-x^2)^{\frac{1}{Log(1-x)}}$ [B.Tech. May/June 2014]

50. Find Taylor's series expansion for $Log\ cosx$ about the point $\frac{\pi}{3}$ [B.Tech. May/June 2014]

UNIT-IV Complex Number

1. Prove that:
   $(1 + cos\theta + i\ sin\theta)^n + (1 + cos\theta - i\ sin\theta)^n = 2^{n+1} cos^n\left(\frac{\theta}{2}\right) \cos\left(\frac{n\theta}{2}\right)$
   (B.tech. Nov/Dec 2012)
2. Find the locus of $|z - 2i| = 3$ (B.tech. Nov/Dec 2012)
3. If n is a positive integer, then prove that :
   $(1+i)^n + (1-i)^n = \left(\sqrt{2}\right)^{n+2} \cos\left(\frac{n\pi}{4}\right)$ (B.tech. Nov/Dec 2012)
4. Solve the equation: $x^9 - x^5 + x^4 - 1 = 0$ (B.tech. Nov/Dec 2012)
5. Find what 'Amplitude of $\frac{z-1}{z+1}$ is $\frac{\pi}{4}$' represent. (B.tech. Nov/Dec 2008)
6. Show that the roots of $(x+1)^6 + (x-1)^6 = 0$ are given by: $-i\cot\left[\frac{(2m+1)\pi}{12}\right]$
   where $0 \le m \le 5$ (B.tech. Nov/Dec 2008)
7. If $u = \frac{z+i}{z+2}$ and $z = x + iy$ then show that, locus of (x, y) is a circle if u is purely imaginary find the center and radius of the circle. (B.tech. Nov/Dec 2010)
8. If $z = x + iy$ where x and y are real, show that when $\frac{z+i}{z+2}$ is real, the locus of the point (x, y) is a straight line. (B.tech. Nov/Dec 2007)
9. If $z = x + iy$ and x, y are real, if $\frac{z+i}{z+2}$ is real, then find the locus of (x, y)
   (B.tech. Nov/Dec 2009,12)
10. If $x_n + iy_n = (1 + i\sqrt{3})^n$, then prove that $x_{n-1}y_n - x_n y_{n-1} = 4^{n-1}\sqrt{3}$
    (B.tech. May/Jun 2009,12)
11. Determine the region in the z-plane represented by:
    (i) $R\left(\frac{1}{z}\right) < \frac{1}{2}$ (ii) $I(z) > 1$ (B.tech. Nov/Dec 2009)
12. Determine the region in the z-plane represented by:
    (i) $R(z^2) \le 1$ (ii) $\frac{\pi}{3} < amp(z) < \frac{\pi}{2}$ (B.tech. May/Jun 2010)

13. Solve the equation: $x^8 + x^5 + x^3 + 1 = 0$ (B.tech. Nov/Dec 2009)
14. Solve the equation: $x^7 + x^4 + x^3 + 1 = 0$ (B.tech. May/Jun 2010,2012)
15. Using De-moivre's theorem solve the equation:
    $(x-1)^{5} + x^5 = 0$ (B.tech. Nov/Dec 2008)
16. Solve the equation: $x^7 - x^4 + x^3 - 1 = 0$ (B.tech. Nov/Dec 2007)
17. If $|z_1 + z_2| = |z_1 - z_2|$ prove that: amp$\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ where z_1 and z_2 are any two complex numbers. (B.tech. Nov/Dec 2007)
18. Determine the region in the z-plane represented by $1 < |z + 2i|$ (B.tech. Apr/May 2012)
19. Find cube roots of unity. (B.tech. Apr/May 2012)
20. For a complex number $= 1 + i$, $|z| =$…….. and $amp(z) =$…….**(F.E.May/Jun2013)**
21. If z_1 and z_2 be two complex numbers such that $|z_1 + z_2| = |z_1 - z_2|$ prove that the difference of their amplitude is $\frac{\pi}{2}$. **(F.E.May/Jun2013,09)**
22. If n is a positive integer, then prove that :
    $(1+i)^{n} + (1-i)^{n} = (\sqrt{2})^{n+2} \cos\left(\frac{n\pi}{4}\right)$ **(F.E.May/June2013)**
23. Prove that: $(x+iy)^{m/n} + (x-iy)^{m/n} = 2(x^2+y^2)^{m/2n} \cos\left[\frac{m}{n} \tan^{-1}\left(\frac{y}{x}\right)\right]$ **(F.E.Nov/Dec2009)**
24. If $\arg(z+1) = \frac{\pi}{6}$ and $\arg(z-1) = \frac{2\pi}{3}$ find z. **(F.E.April/May2012)**
25. Find the roots of the polynomial:
    $x^4 - x^3 + x^2 - x + 1 = 0$ **(F.E.April/May2012)**
26. Prove that: $\left[\frac{1+sin\alpha+icos\alpha}{1+sin\alpha-icos\alpha}\right]^{n} = \cos\left[\frac{n\pi}{2} - n\alpha\right] + isin\left[\frac{n\pi}{2} - n\alpha\right]$ **(F.E.Nov/Dec2008)**
27. Find the different values of $(1+i)^{1/3}$ **(F.E.May/June2008)**
28. Find the locus represented by: $|z + 2 - i| = 4$ **(F.E.May/June2008)**
29. Solve by using complex number:
    $x^{10} + 11x^5 + 10 = 0$ **(F.E.Nov/Dec2007)**
30. Prove that the sum of n^{th} roots of unity is zero. **(F.E.Nov/Dec2007)**

31. Prove that: $coth^{-1}x = \frac{1}{2} Log\frac{x+1}{x-1}$, $|x| > 1$ [B.tech.Old Nov/Dec 2012]
32. If $\cosh(u+iv) = x + iy$ prove that: $\frac{x^2}{cosh^2u} + \frac{y^2}{sinh^2u} = 1$, $\frac{x^2}{cos^2u} - \frac{y^2}{sin^2u} = 1$ [B.tech.Old Nov/Dec 2012]
33. If $i^{\alpha+i\beta} = \alpha + i\beta$ prove that: $\alpha^2 + \beta^2 = e^{-\pi\beta(4n+1)}$ [B.tech.Old Nov/Dec 2012]
34. If $\tan(\theta + i\Phi) = tan\alpha + i\, sec\alpha$ prove that:
    $e^{2\phi} = cot\frac{\alpha}{2}$, $2\theta = n\pi + \frac{\pi}{2} + \alpha$ [B.tech. Nov/Dec 2012]
35. Prove that: $Log\left[\frac{\cos(x-iy)}{\cos(x+iy)}\right] = 2i\, tan^{-1}(tanx\, tanhy)$ [B.tech. Nov/Dec 2012]
36. Prove that: $tanh^{-1}x = sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ [B.tech. Nov/Dec 2012]
37. Prove that: $sech^{-1}[sin\alpha] = logcot\left(\frac{\alpha}{2}\right)$ [B.tech. May/June 2008]

38. Prove that: $Log\left[\frac{(a-b)+i(a+b)}{(a+b)+i(a-b)}\right] = i[\,2n\pi + tan^{-1}\frac{2ab}{a^2-b^2}\,]$ [B.tech. May/June 2008]

39. Prove that: $sinh^{-1}x = Log(x + \sqrt{x^2+1}\,)$ [B.tech.May/June 2008]

40. If $\sin(\theta + i\Phi) = e^{i\alpha}$ prove that: $cos^2\theta = \pm\, sin\alpha$ [B.tech. May/June 2007]

41. Separate into real and imaginary parts: sec $(x + iy)$ [B.tech. May/June 2007]

42. If $\tan(A + iB) = \alpha + i\beta$ show that: $\frac{1-(\alpha^2+\beta^2)}{1+(\alpha^2+\beta^2)} = \frac{cos2A}{cosh2B}$ [B.tech. May/June 2009]

43. Find principal value of $\frac{(a+ib)^{p+iq}}{(a-ib)^{p-iq}}$ [B.tech. May/June 2009]

44. If $x = 2sin\alpha\, cosh\beta$, $y = 2cos\alpha\, sinh\beta$ show that:

(i) $cosec(\alpha - i\beta) + cosec(\alpha + i\beta) = \frac{4x}{x^2+y^2}$

(ii) $cosec(\alpha - i\beta) - cosec(\alpha + i\beta) = \frac{i4y}{x^2+y^2}$ [B.tech. Nov/Dec 2009]

45. Prove that: $i^{a} = \cos\left[\left(2m+\frac{1}{2}\right)\pi\alpha\right] + i\, sin\left[\left(2m+\frac{1}{2}\right)\pi\alpha\right]$ [B.tech. Nov/Dec 2009]

46. Prove that: $Logtan\left(\frac{\pi}{4} + \frac{x}{2}\, i\right) = i\, tan^{-1}(sinhx)$ [B.tech. May/June 2010]

47. Prove that :

(i) $tanh^{-1}x = sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

(ii) $sinh^{-1}x = \frac{1}{2}\, cosech^{-1}\frac{1}{2x\sqrt{1-x^2}}$ [B.tech.Nov/Dec 2008]

48. Prove that: $\tan\left[i\, log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2-b^2}$ [B.tech.Nov/Dec 2008]

49. Separate $tan^{-1}(x + iy)$ into real and imaginary parts. [B.tech.Nov/Dec 2008]

50. Prove that: $tanh^{-1}x = \frac{1}{2}\, Log\frac{1+x}{1-x}$ [B.tech.Nov/Dec 2007]

51. If $\tan(\alpha + i\beta) = x + iy$ show that: $x^2 + y^2 + 2x\, cot2\alpha = 1$ [B.tech. May/June 2012]

52. Prove that: $i^{i} = e^{-\left(2n+\frac{1}{2}\right)\pi}$ [B.tech. May/June 2012]

53. Separate into real and imaginary parts of $cos^{-1}\left(\frac{3i}{4}\right)$ [B.tech. May/June 2012]

54. Separate into real and imaginary parts sin $(x - iy)$ [B.tech. Nov/Dec 2012]

55. If $\cosh(\alpha + i\beta) = a + ib$ then $a =$……. and $b =$…… [F.E. May/June 2013]

56. Show that: $64\, sin^7\theta = 35sin\theta - 21\, sin3\theta + 7\, sin5\theta - sin7\theta$ [F.E. May/June 2013]

57. Prove that: $Log\left(\frac{1}{1-e^{i\theta}}\right) = Log\left(\frac{1}{2}\, cosec\frac{\theta}{2}\right) + i\,\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$ [F.E. May/June 2013]

58. Prove that: $sin7\theta = 7sin\theta - 56sin^3\theta + 112sin^5\theta - 64\, sin^7\theta$ [F.E.Nov/Dec 2009]

59. If $\sin(\alpha + i\beta) = x + iy$ prove that: $\frac{x^2}{cosh^2\beta} + \frac{y^2}{sinh^2\beta} = 1$ [F.E.Nov/Dec 2009]

60. Expand $cos3\theta$ using De-Moivre's theorem. [F.E.April/May 2012]

61. Find the value of a and b if $a + ib = i^{(1+i)}$ [F.E.April/May 2012]

62. Expand $sin^5\theta$ in a series of sines multiples of θ. [F.E.April/May 2012]

63. Prove that the real part of the principal values of $i^{\log(1+i)}$ is $e^{-\pi^2/8}\cos\left(\frac{\pi}{4}\, log2\right)$

64. Show that: $\sqrt[n]{x+iy} + \sqrt[n]{x-iy}$ has n real values and find those of

$\sqrt[3]{1+i\sqrt{3}}+\sqrt[3]{1-i\sqrt{3}}$ [F.E.Nov/Dec 2008]

65. Prove that:

$128[1-cos^{2}\theta]^{4}=cos8\theta-8\,cos6\theta+28\,cos4\theta-56\,cos2\theta+35$

[F.E.Nov/Dec 2008]

66. If $\cos(x+iy)=e^{i\alpha}$ show that: $cosh2y+cos2x=2$ [F.E. May/June 2008]

67. Simplify :$\left[\frac{1+\cos\left(\frac{\pi}{9}\right)+i\sin\left(\frac{\pi}{9}\right)}{1+\cos\left(\frac{\pi}{9}\right)-i\sin\left(\frac{\pi}{9}\right)}\right]^{18}$ [F.E.Nov/Dec 2007]

68. Expand $sin^{5}\theta\, cos^{3}\theta$ in a series of sines multiples of θ. [F.E.Nov/Dec 2007]

UNIT-V Partial Differentiation

1. If $x=\frac{cos\theta}{u}$, $y=-\frac{sin\theta}{u}$ and $z=f(x,y)$ show that :
   $\left(\frac{\partial x}{\partial u}\right)_{\theta}\left(\frac{\partial u}{\partial x}\right)_{y}=cos^{2}\theta$ [B.Tech.(Old)Nov/Dec 2012]
2. If $z(x+y)=x^2+y^2$ show that : $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^{2}=4\left[1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right]$
3. If $u=tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that :
   $x^2\frac{\partial^2 u}{\partial x^2}+2xy\,\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=sin2u\,(1-4sin^{2}u)$ [B.Tech.(Old)Nov/Dec 2012]
4. If $x=uv, y=\frac{u}{v}$, $z=f(x,y)$ prove that $\frac{\partial z}{\partial x}=\frac{1}{2v}\,\frac{\partial z}{\partial u}+\frac{1}{2u}\,\frac{\partial z}{\partial v}$ [B.Tech.Nov/Dec 2012]
5. If $z=x^2+y^2$, $x=at\,sint$, $y=at\,cost$ find $\frac{dz}{dt}$ [B.Tech.(Old)Nov/Dec 2012]
6. If $\theta=t^n\,e^{-\frac{r^2}{4t}}$ find $\frac{\partial\theta}{\partial t}$. [B.Tech.Nov/Dec 2012]
7. Find $\frac{dy}{dx}$ if $y^x+x^y=(x+y)^{x+y}$ [B.Tech.Nov/Dec 2012]
8. If $u=ax+by$, $v=bx-ay$ find the value of $\left(\frac{\partial x}{\partial u}\right)_{v}\left(\frac{\partial y}{\partial v}\right)_{u}$
9. If $Log(sinu)=\frac{(x^2+y^2)^{3/2}}{x+y}$ find the value of $x\,\frac{\partial u}{\partial x}+y\,\frac{\partial u}{\partial y}$ [B.Tech.Nov/Dec 2012]
10. If $x=e^{\theta+i\phi}$, $y=e^{\theta-i\phi}$ prove that $J\,J'=1$ [B.Tech.Nov/Dec 2012]
11. If $z=f(x,y)$, $x=u\,cos\alpha-v\,sin\alpha$, $y=u\,sin\alpha+v\,cos\alpha$ prove that
    $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}$ [B.Tech.Nov/Dec 2008]
12. If $2x=r[e^{\theta}+e^{-\theta}]$, $2y=r[e^{\theta}-e^{-\theta}]$ then prove that $\left(\frac{\partial x}{\partial r}\right)_{\theta}=\left(\frac{\partial r}{\partial x}\right)_{y}$
13. Find $\frac{du}{dx}$, given that: $u=x\,Logxy$ and $x^3+y^3=-3xy$ [B.Tech.Nov/Dec 2008]
14. If $x^x y^y z^z=c$ when $x=y=z$ Prove that $\frac{\partial^2 z}{\partial x\,\partial y}=-[x\,(Logex)]^{-1}$

    If $z=f(x,y)$ where $x=e^u+e^{-v}$, $y=e^{-u}-e^{v}$ then prove that :
    $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\,\frac{\partial z}{\partial x}-y\,\frac{\partial z}{\partial y}$ [B.Tech.Nov/Dec 2008]

15. If $u = sin^{-1}\left[\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right]$ show that: $u_x + \frac{y}{x}\, u_y = 0$ [B.Tech.Nov/Dec 2008]

16. If $u = \frac{x+y}{1-xy}$, $v = tan^{-1}x + tan^{-1}y$ find $\frac{\partial(u,v)}{\partial(x,y)}$ [B.Tech.Nov/Dec 2008]

17. If $\tan^2\theta = \frac{y}{x}$, show that $\left(\frac{\partial\theta}{\partial x}\right)_y \left(\frac{\partial\theta}{\partial y}\right)_x = \frac{-1}{4(x+y)^2}$ [B.Tech.Nov/Dec 2010]

18. If $u = cosec^{-1}\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$ prove that :

$x^2u_{xx} + 2xy\, u_{xy} + y^2u_{yy} = \frac{tanu}{12}\left[\frac{13}{12} + \frac{tan^2u}{12}\right]$ [B.Tech.Nov/Dec 2010]

19. If $u(x+y) = x^2 + y^2$ show that : $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left[1 - \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\right]$

20. If $\theta = t^n\, e^{-\frac{r^2}{4t}}$ then find the value of n which will make

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$ [B.Tech.Nov/Dec 2010]

21. If $u = Log(x^3 + y^3 - x^2y - xy^2)$ prove that:

$x^2\frac{\partial^2u}{\partial x^2} + 2xy\,\frac{\partial^2u}{\partial x\,\partial y} + y^2\frac{\partial^2u}{\partial y^2} = -3$ [B.Tech.May/June 2007]

22. Find $\frac{dz}{dt}$ when $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$. Verify by direct substitution.

23. Find the stationary values of : $x^3y^2(1-x-y)$ [B.Tech.Nov/Dec 2009]

24. Find $\frac{du}{dx}$ if $u = tan^{-1}\left(\frac{x}{y}\right)$, x and y are related by $x^2 + y^2 = a^2$

25. If $u = \sin(\sqrt{x} + \sqrt{y})$ prove that $x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y} = \frac{1}{2}\,(\sqrt{x} + \sqrt{y})\cos(\sqrt{x} + \sqrt{y})$

26. If $r^2 = x^2 + y^2 + z^2$ and $u = r^m$ prove that :

$\frac{\partial^2u}{\partial x^2} + \frac{\partial^2u}{\partial y^2} + \frac{\partial^2u}{\partial z^2} = m(m+1)r^{m-2}$ [B.Tech.Nov/Dec 2009]

27. If $x = r\,cosh\theta$, $y = r\,sinh\theta, z = f(x,y)$ prove that:

$(x-y)\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right) = r\frac{\partial z}{\partial r} - \frac{\partial z}{\partial\theta}$ [B.Tech.Nov/Dec 2009]

28. If $u = sin^{-1}\left(\frac{x^{1/4}-y^{1/4}}{x^{1/5}+y^{1/5}}\right)$ prove that

$x^2\frac{\partial^2u}{\partial x^2} + 2xy\,\frac{\partial^2u}{\partial x\,\partial y} + y^2\frac{\partial^2u}{\partial y^2} = \frac{tanu}{400}\,[tan^2u - 19]$ [B.Tech.May/June 2010]

29. Find $\frac{dz}{dt}$ if $z = xy^3 + x^3y$, $x = at^2$, $y = 2at$ [B.Tech.May/June 2010]

30. If $f = x^2\,tan^{-1}\left(\frac{y}{x}\right) - y^2\,tan^{-1}\left(\frac{x}{y}\right)$ show that :

$\frac{\partial^2 f}{\partial x\partial y} = \frac{x^2-y^2}{x^2+y^2}$ [B.Tech.May/June 2010]

31. If $z = f(u,v)$ and $u = e^{\theta}\,cos\phi$, $v = e^{\theta}sin\phi$ show that

$\frac{\partial^2z}{\partial\theta^2} + \frac{\partial^2z}{\partial\phi^2} = (u^2 + v^2)\left[\frac{\partial^2z}{\partial u^2} + \frac{\partial^2z}{\partial v^2}\right]$ [B.Tech.May/June 2010]

32. If $u = a\,coshx\,cosy$, $v = a\,sinhx\,siny$ show that

$\frac{\partial(u,v)}{\partial(x,y)} = \frac{a^2}{2}[cosh2x - cos2y]$ [B.Tech.Nov/Dec 2008]

33. If $x^2 = au + bv$, $y^2 = au - bv$ then prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u = \frac{1}{2}$

34. If $u = \sin\left(\frac{x}{y}\right)$ where $x = e^t$, $y = t^2$ find $\frac{du}{dt}$ [B.Tech.Nov/Dec 2007]

35. Find the value of n so that $u = r^n\ [3cos^{\,2}\theta - 1]$ satisfies the partial differential equation $\frac{\partial}{\partial r}\left[r^2\ \frac{\partial u}{\partial r}\right] + \frac{1}{sin\theta}\ \frac{\partial}{\partial \theta}\ \left[sin\theta\ \frac{\partial u}{\partial \theta}\right] = 0$ [B.Tech.Nov/Dec 2007]

36. If $u = sinh^{\,-1}\left(\frac{x^3+y^3}{x+y}\right)$ find the value of $x^2\frac{\partial^{\,2}u}{\partial x^{\,2}} + 2xy\ \frac{\partial^{\,2}u}{\partial x\,\partial y} + y^2\frac{\partial^{\,2}u}{\partial y^{\,2}}$

37. If $z = e^{xy^2}$, $x = t\ cost, y = t\ sint$ find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$ [B.Tech.Nov/Dec 2007]

38. If $u = f_1(x,y)$ where $x = f_2(t), y = f_3(t)$ then $\frac{du}{dt} = \cdots$ [F.E.2013]

39. If $u = x^2tan^{\,-1}\left(\frac{y}{x}\right)$ then $x^2\frac{\partial^{\,2}u}{\partial x^{\,2}} + 2xy\ \frac{\partial^{\,2}u}{\partial x\,\partial y} + y^2\frac{\partial^{\,2}u}{\partial y^{\,2}}\ = \cdots$ [F.E.2013]

40. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^{\,2} u = -\frac{9}{(x+y+z)^{\,2}}$

41. If $u = 2xy, v = x^2 - y^2, x = r\ cos\theta, y = r\ sin\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$ [F.E.2013]

42. If $u = sin^{\,-1}(x^2 + y^2)$ find the value of $x^2\frac{\partial^{\,2}u}{\partial x^{\,2}} + 2xy\ \frac{\partial^{\,2}u}{\partial x\,\partial y} + y^2\frac{\partial^{\,2}u}{\partial y^{\,2}}$

43. If $u = sin^{\,-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that: $x^2\frac{\partial^{\,2}u}{\partial x^{\,2}} + 2xy\ \frac{\partial^{\,2}u}{\partial x\,\partial y} + y^2\frac{\partial^{\,2}u}{\partial y^{\,2}} = -\frac{sinu\ cos2u}{4cos^{\,3}u}$

44. If $u = f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ [F.E.2009]

45. If $z = f(x,y),\ u = lx + my\ , v = ly - mx$ show that $\frac{\partial^{\,2}z}{\partial x^{\,2}} + \frac{\partial^{\,2}z}{\partial y^{\,2}} = (l^2 + m^2)\left[\frac{\partial^{\,2}z}{\partial u^{\,2}} + \frac{\partial^{\,2}z}{\partial v^{\,2}}\right]$ [F.E.2009]

46. If $u = x^2y^2\ , x = 2at\ , y = at^3$ find $\frac{du}{dt}$ [F.E.2012]

47. If $u = sin^{\,-1}\left(\frac{x}{y}\right) + tan^{\,-1}\left(\frac{y}{x}\right)$ then $x\ \frac{\partial u}{\partial x} + y\ \frac{\partial u}{\partial y} = \cdots$ [F.E.2012]

48. $f(x,y)$ has a maximum value at (a,b) if …….. [F.E.2012]

49. If $u = \frac{y}{z} + \frac{z}{x}$ show that: $x\ \frac{\partial u}{\partial x} + y\ \frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ [F.E.2012]

50. If $u + x + y + z\ , uv = y + z,\ uvw = z$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$ [F.E.2012]

51. If $x = \frac{vw}{u}\ , y = \frac{wu}{v}\ , z = \frac{uv}{w}$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = 4$ [F.E.2008]

52. If $x = \sqrt{vw}\ , y = \sqrt{wu}\ , z = \sqrt{uv}$ and $u = rsin\theta\ cos\phi, v = rsin\theta\ sin\phi, w = rcos\theta$ find the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ [F.E.2008]

53. If $y = x\ cosu$, then find the value of

    (i)$x\ \frac{\partial u}{\partial x} + y\ \frac{\partial u}{\partial y}$

    (ii) $x^2\frac{\partial^{\,2}u}{\partial x^{\,2}} + 2xy\ \frac{\partial^{\,2}u}{\partial x\,\partial y} + y^2\frac{\partial^{\,2}u}{\partial y^{\,2}}$ [F.E.2007]

54. If $ux + vy = 0,\ \frac{u}{x} + \frac{v}{y} = 1$ prove that $\left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial v}{\partial y}\right)_x = \frac{x^2+y^2}{y^2-x^2}$ [F.E.2007]

UNIT-VI Maxima and Minima

1. State necessary condition for existence of maxima and minima for $u = f(x, y)$.
2. $f(x, y)$ has minimum value at (a,b) if.............. [F.E.2013]
3. The stationary points of the function: $f(x, y) = 3x^2 - y^2 + x^3$ will be.........
4. Find stationary values of: $x^3 + y^3 - 3axy,\ \ a > 0$
   [B.Tech.Nov/Dec2008]
5. Examine for minimum and maximum values of : $x^2 + y^2 + (10 - x - y)^{2}$
6. Find the stationary values of : $x^3y^2(1 - x - y)$ [B.Tech.Nov/Dec 2009]
7. Find maxima and minima of : $xy + \frac{a^3}{x} + \frac{a^3}{y}, a > 0.$ [B.Tech.May/June 2010]
8. Discuss maxima and minima of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$
9. Examine the minimum and maximum value of : $sinx + siny + \sin(x + y)$.
10. Discuss the maxima and minima of : $x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$ [F.E.2008]
11. If $x = u(1 - v), y = uv$ then prove that: $J\,J' = 1$.
    [B.Tech.(Old)Nov/Dec 2012]
12. If $x = e^{\theta + i\phi}, y = e^{\theta - i\phi}$ prove that $J\,J' = 1$ [B.Tech.Nov/Dec 2012]
13. If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$ prove that : $J\,J' = 1$
14. Show that: $\frac{\partial(r,s,t)}{\partial(x,y,z)} = x^2y$ use the following information:
    $x = r + s + t, xy = s + t, xyz = t$ [B.Tech.Nov/Dec 2010]
15. If $x + y = 2e^{\theta} \cos\phi, x - y = 2i\, e^{\theta} \sin\phi$ prove that: $J\,J' = 1$[B.Tech.Nov/Dec 2009]
16. If $x = uv, y = \frac{u}{v}$ prove that: $J\,J' = 1$ [B.Tech.Nov/Dec 2009]
17. If $x = e^{\phi} \sec\theta, y = e^{\phi} \tan\theta$ then prove that: $J\,J' = 1$ [B.Tech.May/June 2010]
18. If $x = r\cos\theta, y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} = \cdots$ [F.E.2013]
19. If $x = e^{\alpha}\cos\beta, y = e^{\alpha}\sin\beta$ then show that $JJ' = 1$ [F.E.2008]
20. If $x = \frac{vw}{u}, y = \frac{wu}{v}, z = \frac{uv}{w}$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = 4$ [F.E.2008]
21. If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi, v = r\sin\theta\sin\phi,$
    $w = r\cos\theta$ find the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ [F.E.2008]
22. If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ find the Jacobian of the given transformation.

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