Unit-I Function of complex variable

1. If f(z) is analytic in a closed curve c,except at a finite number of singular points within c, then: $\int_c f(z)dz =$ ……. [B.tech.Nov/Dec 2012]
2. The value of $\int_0^{\pi i} \cos z\, dz =$ …….. [B.tech.Nov/Dec 2012]
3. If f(z) has a pole of order three at $z = a$ then: $\text{Res}f(a) = \cdots$ [B.tech.Nov/Dec 2012]
4. If C is the circle $|z| = 2$ then $\int_c \frac{1}{z-1} dz =$ …… [B.tech.Nov/Dec 2012]
5. Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2+y^2}$ are harmonic functions of (x,y) but not harmonic conjugates. [B.tech.Nov/Dec 2012]
6. Evaluate: $\int_c \frac{2z^2+z}{z^2-1} dz$ where c is $|z - 1| = 1$ by Cauchy integral formula. [B.tech.Nov/Dec 2012]
7. Evaluate $\int_c |z|^2 dz$ around the square with vertices at (0,0),(1,0),(1,1) and (0,1). [B.tech.Nov/Dec 2012]
8. Find the image of the circle $|z| = 2$ under the transformation $w = z + 3 + 2i$. [B.tech.Nov/Dec 2012]
9. If f(z) is analytic within and on a closed curve and if 'a' is any point within c, then $\frac{1}{2\pi i} \int_c \frac{f(z)}{z-a} =$…….. [B.tech. May/June 2012]
10. The value of $\int_0^{2i} \sinh z\, dz =$……. [B.tech. May/June 2012]
11. The poles of the function $f(z) = \frac{z^3-1}{z^3+1}$ are ….. [B.tech. May/June 2012]
12. If C is the circle $|z| = 2$ then $\int_c \frac{e^z}{(z-3)^2} dz =$…… [B.tech. May/June 2012]
13. Let $f(z) = u + iv$ be an analytic function, if $u = 3x - 2xy$ then find v and express f(z) in terms of z. [B.tech. May/June 2012]
14. Evaluate: $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 2$ by Cauchy integral formula. [B.tech. May/June 2012]
15. Find the image of the lines :
    i) $x = y + 1$
    ii) The line joining A (1+i) to B (2+3i) in z-plane under the transformation $W=\frac{i}{z}$
    [B.tech. May/June 2012]
16. Evaluate: $\int_0^\infty \frac{dx}{(1+x^2)^2}$ using residue theorem.
17. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function at $z = x + iy$, find f (z) in terms of z.
18. Evaluate: $\int_c \frac{z}{(z-1)(z-3)} dz$ where c is the circle $|z| = 3$ [B.tech. May/June 2011]

19. If f(z) is analytic at all points within C in a multiple connected region R then, $\int_c f(z)dz = \ldots\ldots.$ [B.tech. May/June 2012]

20. If C is the circle $|z| = \frac{1}{2}$ then $\int_c \frac{e^{-z}}{z+1} dz$ =……. [B.tech. May/June 2011]

21. If C is the circle $|z| = 1$ then $\int_c \frac{e^{-z}}{z^2} dz$ =……. [B.tech. May/June 2011]

22. Evaluate: $\int_c \frac{3z^2+z}{z^2-1}$ dz where c is $|z - 1| = 1$ [B.tech. May/June 2011]

23. Cauchy-Riemann equation in Cartesian coordinates are……
[B.tech. Oct/Nov 2011]

24. If f(z) is analytic function and $f'(z)$ is continuous at each point within and on a closed curve C,then $\int_c f(z)dz = \ldots\ldots.$ [B.tech.Oct/Nov 2011]

25. If $f'(z) = \cos x \cosh y - i \sin x \sinh y$ then $f(z) =$ ……..
[B.tech.Oct/Nov 2011]

26. If C is the circle $|z - 1| = 1$ then $\int_c \frac{\cos z}{z} dz$ =……. [B.tech.Oct/Nov 2011]

27. Find p such that the function: $f(z) = r^2\cos 2\theta + ir^2 \sin p\theta$ is analytic.
[B.tech.Oct/Nov 2011]

28. Find the analytic function: $f(z) = u + iv$ given $r = a(1 + \cos\theta)$

29. Evaluate: $\int_c \frac{e^z}{\cos\pi z} dz$ where C is the circle $|z| = 1$ [B.tech. May/June 2010]

30. If f (z) is analytic within and on a closed curve C then $\int_c \frac{f(z)}{z-a} dz$ =…….
[B.tech. May/June 2010]

31. If f (z) is a ingle valued and analytic within and on a closed curve C then $\int_c \frac{f(z)}{(z-a)^{n+1}} dz$ =……. [B.tech. May/June 2010]

32. Every analytic function $f(z) = u(x, y) + iv(x, y)$ defines two families of curves $u(x, y) = C_1$ and $v(x, y) = C_2$ .These family of curves are……….to each other.
[B.tech. May/June 2010]

33. Prove that the function $\cosh z$ is analytic and find its derivative.
[B.tech. May/June 2010]

34. If C is the circle $|z| = 2$ then evaluate $\int_c \frac{e^{-z}}{z+1} dz$ [B.tech. May/June 2010]

35. The necessary and sufficient condition for the function $w = f(z) = u(x, y) + iv(x, y)$ to be analytic in a region R, are……..
[B.tech. May/June 2010]

36. An electrostatic field in the xy plane is given by the potential function : $\phi = 3x^2y - y^3$. Find the stream function. [B.tech. May/June 2011]

37. Evaluate: $\int_c \frac{2z-1}{z(z+1)(z-3)} dz$ where c is the circle$|z| = 2$ [B.tech. May/June 2010]

38. If C is the circle $|z - 1| = 3$ then $\int_c \frac{\cos z}{z-\pi} dz$ =……. [B.tech. May/June 2010]

39. Determine p such that the function $f(z) = \frac{1}{2} \operatorname{Log}(x^2 + y^2) + i\tan^{-1}\frac{px}{y}$
is analytic function. [B.tech.(Old) May/June 2010]

40. Determine analytic function $w = u + iv$ if $v = \log(x^2 + y^2) + x - 2y$
[B.tech.(Old) May/June 2010]
41. Cauchy-Riemann equation in polar coordinates are……. [B.tech. Oct/Nov 2009]
42. C-R equation in Cartesian form = …….. [B.tech. Oct/Nov 2009]
43. Residue of $e^{\frac{1}{z^2}} \cos z =$……. [B.tech. Oct/Nov 2009]
44. Residue of $\frac{\tan z}{z}$ at $z = 0$ is…… [B.tech. Oct/Nov 2009]
45. If f(z) has a pole of order m at $z = a$ then the residue of f(z) at $z = a$ is……
[B.tech. Oct/Nov 2009]
46. Prove that an analytic function with constant amplitude is constant.
[B.tech. Oct/Nov 2009]
47. Evaluate: $\int_c \frac{e^{3z}}{z-\pi i} dz$ where C is the ellipse $|z - 2| + |z + 2| = 6$
[B.tech. Oct/Nov 2009]
48. If $f(z) = u + iv$ analytic function, find f (z) if $u + v = r^2(\cos 2\theta + \sin 2\theta)$ when f (0) =0. [B.tech. Oct/Nov 2009]
49. Evaluate using residue theorem $\int_C \frac{z-3}{z^2+2z+5} dz$ where C:$|z + 1 - i| = 3$.
[B.tech. Oct/Nov 2009]
50. Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle $|z - 2i| = \frac{3}{2}$
[B.tech.May/June 2009]
51. If $u + v = e^x(\cos y + \sin y) + \frac{x-y}{x^2+y^2}$, find the analytic function $f(z) = u + iv$
Where f(1)=1. [B.tech.May/June 2009]
52. Evaluate: $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2}$ [B.tech. May/June 2009]
53. Find Laurent's expansion for $f(z) = \frac{1}{(1-z^2)(z+2)}$
For i) $|z| < 1$ ii) $1< |z|<2$ [B.tech. May/June 2009]
54. Find an analytic function f(z) such that
$\text{Re}[f'(z)] = 3x^2 - 4y - 3y^2$ and $f(1 + i) = 0$ [S.E.Oct/Nov 2011]
55. Evaluate: $\int_1^i \frac{\text{Log}^3 z}{z} dz$ along the arc of the circle $|z| = 1$ [S.E.Oct/Nov 2011]
56. Prove that $u = r^2\cos 2\phi$ is harmonic function and find its harmonic conjugate and also corresponding analytic function. [S.E.Oct/Nov 2011]
57. Evaluate $\int_C \frac{\sin z}{(z-1)^2(z^2-9)} dz$ where C is the circle :
(i) $|z - 3i| = 1$ (ii) $|z - 3| = \frac{1}{2}$ [S.E.Oct/Nov 2011]
58. If $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ find Laurent's series expansion in
(i) $0 < |z - 1| < 4$ (ii) $|z - 1| > 4$ [S.E.Oct/Nov 2011]
59. Evaluate by Cauchy's residue theorem $\int_C \frac{1}{z \sin z} dz$ where C is the circle $|z| = \frac{1}{2}$

60. Find the image of the square with vertices (0,0) , (2,0) (2,2) (0,2) under the transformation $W = (1+i)z + (2+i)$ [S.E.Oct/Nov 2011]

61. Evaluate: $\int_0^{2\pi} \frac{2}{2+\cos\theta}\, d\theta$ [S.E.Oct/Nov 2011]

62. Find the bilinear transformation which maps the points $1, -1, \infty$ in z-plane on to the points $1, i, -1$ in w-Plane. [S.E.Oct/Nov 2011]

63. Evaluate: $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2}$ $(a > 0)$ [S.E.Oct/Nov 2011]

64. Determine whether the function $f(z) = z\, e^{-z}$ is analytic or not. [S.E.Nov/Dec 2012]

65. Evaluate: $\int_0^{\pi-\pi i} e^{\bar{z}}\, dz$ along the curve C: $x = t, y = -t$ [S.E.Nov/Dec 2012]

66. Prove that: $v = r^2 \sin 2\phi - \frac{1}{r} \sin\phi$ is harmonic and find its harmonic conjugates and also corresponding analytic function. [S.E.Nov/Dec 2012]

67. Evaluate: $\int_C \frac{z^2}{z^4-1} dz$ where C is the circle

(i) $|z| = \frac{1}{2}$ (ii) $|z-1| = \frac{1}{2}$ [S.E.Nov/Dec 2012]

68. Expand $\cos z$ in to a Taylor's series about the point $z = \frac{\pi}{2}$ [S.E.Nov/Dec 2012]

69. Evaluate by Cauchy's residue theorem: $\int_C \frac{z}{\sin^2 z}$ dz where C: $|z| = \frac{1}{5}$ [S.E.Nov/Dec 2012]

70. Find the bilinear transformation which maps the points $2, i, -2$ in z-plane on to the points $1, i, -1$ in w-Plane. [S.E.Nov/Dec 2012]

71. Evaluate: $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$ [S.E.Nov/Dec 2012]

72. Find the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$ [S.E.May/June 2012]

Evaluate: $\int_C \mathrm{Re}(z) dz$ where C is the semi unit circle. [S.E.May/June 2012]

73. Evaluate by using Cauchy's integral formula: $\int_C \frac{z+1}{z^2+2z+4} dz$ where C is $|z+1+i| = 2$ [S.E.May/June 2012]

74. Evaluate: $\oint_C \frac{\coth z}{z-i}$ dz where C is $|z| = 2$ by Cauchy's residue theorem. [S.E.May/June 2012]

75. Expand $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ for $1 < |z+1| < 3$ [S.E.May/June 2012]

76. Find the bilinear transformation which maps the points $z = -1, 2, i-1$ in to the points $W = \infty, \frac{2i}{3}, i-1$. [S.E.May/June 2012]

77. Find the analytic function whose real part is $y + e^x \cos y$ [S.E.Nov/Dec 2008]

78. Evaluate: $\int_C (z - z^2) dz$ where C is upper half of the circle $|z| = 1$ [S.E.Nov/Dec 2008]

79. Apply Cauchy's residue theorem to evaluate: $\oint_C \frac{dz}{(z^2+1)(z^2+4)}$ where C is the circle $|z| = 1.5$ [S.E.Nov/Dec 2008]

80. Determine the bilinear transformation that maps the points: $1-2i, 2+i, 2+3i$ respectively into $2+2i, 1+3i, 4$. [S.E.Nov/Dec 2008]

81. Use Cauchy's integral formula to evaluate: $\int_C \frac{z^2-2z+1}{(z-i)^2} dz$ where C is $|z| = 2$ [S.E.Nov/Dec 2008]

82. Prove that $u = \text{Log}(x^2+y^2)$ is harmonic function and find its harmonic conjugate and also corresponding analytic function. [S.E.Nov/Dec 2008]

83. If the real part of an analytic function $f(z)$ is x^2-y^2-y then find imaginary part. [S.E.Nov/Dec 2014]

84. If $f(z) = \frac{a}{2} r\cos\theta + i(r\sin\theta + 2)$ is harmonic, then find the value of a. [S.E.Nov/Dec 2014]

85. Find the imaginary part of analytic function whose real part is $r^{-4}\cos 4\theta$ [S.E.Nov/Dec 2014]
    Find whether the function $u = \text{Log}|z|^2$ is harmonic. If so find analytic function whose real part is u. [S.E.Nov/Dec 2014]

86. Show that the image of the line $x = 0$ under the transformation $w = e^z$ is a circle.
    Evaluate $\int_0^{1+i} (x+iy)dz$ along $y = x$ [S.E.Nov/Dec 2014]
    Evaluate $\int_C (z-i)dz$ where C is 0 to i [S.E.Nov/Dec 2014]

87. Find the poles of f(z) and residues at the poles which lie on imaginary axis if $f(z) = \frac{z^2-2z}{(z+1)(z^2+4)}$ [S.E.Nov/Dec 2014]

88. Find the Bilinear transformation which maps the points $z = 1, -i, 1$ onto the points $w = 2, i, -2$ [S.E.Nov/Dec 2014]

89. Evaluate $\int_{(0,0)}^{(1,1)} (3x^2+4xy+ix^2)dx$ along $y = x^2$ [S.E.Nov/Dec 2014]

90. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ for $0 < |z-1| < 1$ [S.E.Nov/Dec 2014]

91. Evaluate $\oint_C \frac{dz}{\sinh^2 z}$ where C is $|z| = 2$ by using Cauchy's integral formula.

92. Under the transformation $W = z + \frac{a^2-b^2}{4z}$ maps the circle of radius $\frac{1}{2}(a+b)$ with centre at origin in the z-plane into an ellipse on W-plane.

93. Evaluate $\oint_C \frac{\cosh z}{(z+1)^3(z-1)} dz$ where C is $|z| = 2$ by Cauchy's Residue theorem.

94. Evaluate: $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ using residue theorem. [S.E.Nov/Dec 2014]

Unit-II Application of Complex Variable

1. Evaluate $\int_c |z|^2\, dz$ around the square with vertices at (0,0),(1,0),(1,1) and (0,1). [B.tech.Nov/Dec 2012]

2. Find the image of the lines :

   i) $x = y + 1$

   ii) The line joining A (1+i) to B (2+3i) in z-plane under the transformation $W = \frac{i}{z}$

3. Evaluate: $\int_1^i \frac{\mathrm{Log}^3 z}{z}\, dz$ along the arc of the circle $|z| = 1$

4. Find the image of the square with vertices (0,0) , (2,0) (2,2) (0,2) under the transformation $W = (1 + i)z + (2 + i)$ [S.E.Oct/Nov 2011]

5. Evaluate: $\int_0^{\pi-\pi i} e^{\bar{z}}\, dz$ along the curve C: $x = t, y = -t$ [S.E.Nov/Dec 2012]

6. Find the bilinear transformation which maps the points $2, i, -2$ in z-plane on to the points $1, i, -1$ in w-Plane. [S.E.Nov/Dec 2012]

7. Find the bilinear transformation which maps the points $1, -1, \infty$ in z-plane on to

8. Find the bilinear transformation which maps the points $z = -1, 2, i - 1$ in to the points $W = \infty, \frac{2i}{3}, i - 1$

9. Determine the bilinear transformation that maps the points: $1 - 2i, 2 + i, 2 + 3i$ respectively

   into $2 + 2i, 1 + 3i, 4$.

10. Evaluate $\int_0^{1+i} (x + iy)dz$ along $y = x$

11. Evaluate $\int_C (z - i)dz$ where C is 0 to i

12. Find the Bilinear transformation which maps the points $z = 1, -i, 1$ onto the points

    $w = 2, i, -2$

13. Evaluate $\int_{(0,0)}^{(1,1)} (3x^2 + 4xy + ix^2)dx$ along $y = x^2$

14. Under the transformation $W = z + \frac{a^2-b^2}{4z}$ maps the circle of radius $\frac{1}{2}$ $(a + b)$ with centre at origin in the z-plane into an ellipse on W-plane

Unit-III Vector Integration

1. Find the work done by the force $\overline{F} = (2xy + z^3)i + x^2j + 3xz^2k$ along the curve between $(1, -2, 1)$ to $(3,1,4)$

2. Find the work done by the force $\overline{F} = (2xz^3 + 6y)i + (6x - 2yz)j + (3x^2z^2 - y^2)k$

   along the curve from (0,0,0) to (1,0,0) then (1,0,0) to (1,1,1)

3. Find the work done by the force

   $\overline{F} = 2xyz^2i + [x^2z^2 + z\cos(yz)]j + [2x^2yz + y\cos(yz)]k$

   from (1,0,1) to $\left(0, \frac{\pi}{2}, 1\right)$

4. If $\overline{F} = x^2i - xyj$ then find $\int_{(0,0)}^{(1,1)} \overline{F}\, d\overline{r}$ along the parabola $y = x^2$

5. Find the work done when a force $\overline{F} = x^2i + xyj$ moves a particle in x-y plane from

   $(0,0)$ to $(1,1)$ along the path $y^2 = x$

6. Calculate the work done in moving a particle in force field given by $\overline{F} = x^2y^2i + yj$

   along $x^2 = 4y$ from (0,0) to (4,4)

7. If a force $\overline{F} = 2x^2yi + 3xyj$ displaces a particle in the xy-plane from (0, 0) to (1, 4)

   along a curve $y = 4x^2$ find the work done.

8. Find the work done by a force $\overline{F} = yzi + xzj + xyk$ along

   $x = a\cos t, y = a\sin t, z = at$ from $t = 0$ to $t = \frac{\pi}{4}$

9. Find the work done in moving a particle once round an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ in the

   plane $z = 0$ in the force field given by: $\overline{F} = (3x - 2y)i + (2x + 3y)j + y^2k$

10. Apply Green's theorem to evaluate $\int_C [e^{-x}\sin y\,dx + e^{-x}\cos y\,dy]$ where C is the rectangle whose vertices are $(0,0), (\pi, 0)$, $\left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$

11. Apply Green's theorem to evaluate: $\int_C (xy + y^2)dx + x^2\, dy$ where C is the closed

    curve of the region bounded by $y = x$ and $y = x^2$

12. Apply Green's theorem to evaluate: $\int_C \overline{F}\, d\,\overline{r}$ where $\overline{F} = x^2 i + xyj$ and C is a triangle having vertices A (0, 2), B (2, 0) and C (4, 2)

13. Apply Green's theorem to evaluate $\int_C \overline{F}\, d\,\overline{r}$ where $\overline{F} = (y - \sin x)i + \cos x\, j$ over

$y = 0, x = \frac{\pi}{2}, y = \frac{2x}{\pi}$

14. Apply Green's theorem to evaluate $\int_C \overline{F}.\, d\overline{r}$ for

$\overline{F} = (x^2 - 2xy)i + (xy^2 + 3)j\,,\; y^2 = 8x\,, x = 2$

15. Using Green's theorem evaluate $\int_C \overline{F}\, d\,\overline{r}$ where

$\overline{F} = (2x^2y + 3z^2)i + (x^2 + 4yz)j + (2y^2 + 6xz)k$ and C is the curve enclosing a region $y^2 = 4ax\,, x = a$ in the plane $z = 0$

16. Apply Green's theorem to evaluate: $\int_C \overline{F}\, d\overline{r}$ where $\overline{F} = 2y^2 i + 3xj$ over $y = x^2$ and

$y = x$

17. Apply Stoke's theorem to evaluate $\int_C \overline{F}.\, d\overline{r}$, $\overline{F} = y^2 i + xyj + xzk$ where C is the boundary of the curve of hemisphere $x^2 + y^2 + z^2 = a^2$, $z > 0$ in positive direction.

18. Apply Stoke's theorem to evaluate $\int (x^2 + y - 4)dx + 3xydy + (2xz + z^2)dz$

Over the surface of the hemisphere $x^2 + y^2 + z^2 = 9$

19. Apply Stokes-theorem evaluate $\int_C [(x + 2y)dx + (x - z)dy + (y - z)dz]$ where C is the boundary of the triangle with vertices (2, 0, 0) , (0 , 3, 0) and (0, 0, 6).

20. Apply stokes theorem to evaluate $\int_C \overline{F}.\, d\overline{r}$ for

$\overline{F} = 2y(1 - x)i + (x - x^2 - y^2)j + (x^2 + y^2 + z^2)k$ over the area of the triangle

$x + y + z = 2$

21. Evaluate: $\iint_S (\nabla \times \overline{F}).\, ds$ where $\overline{F} = (x^3 - y^3)i - xyzj + y^3 k$ and S is the surface

$x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$

22. Evaluate: $\iint_S \overline{F}\, \hat{n}\, ds$ as a volume integral if $\overline{F} = x^3 i + y^3 j + z^3 k$ over $x^2 + y^2 + z^2 = a^2$

23. Evaluate $\iint_S (x^3dydz + x^2ydzdx + x^2zdxdy)$ where S surface bounded by the cylinder $z = 0 , z = 3$ & $x^2 + y^2 = 4$

24. Evaluate: $\iint_S (x^3i + y^3j + z^3k)ds$ where $x^2 + y^2 + z^2 = 16$.

25. Evaluate: $\iint_s \overline{F}\, \hat{n}\, ds$ where $\overline{F} = 4xi - 2y^2j + z^2k$ and S is the region bounded by $y^2 = 4x , x = 1 , z = 0 , z = 3$

26. Evaluate: $\iint_s \overline{F}\, \hat{n}\, ds$ where $\overline{F} = (2xy + z)i + y^2j - (x + 3y)k$ over the volume

bounded by $2x + 2y + z = 6 , x = 0, y = 0, z = 0.$

Unit-IV Application of partial differential equation

1. The solution of $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y}$ is u =……
2. The solution of $\frac{\partial^2 z}{\partial x\,\partial y} = \cot x \cos y$ is z……..
3. The solution of : $\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t} + u$ is u =…….
4. If $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends , find the solution with conditions :
   $y(0, t) = y(L, t) = 0, y(x, 0) = y_0 \sin\frac{\pi x}{L}$ and $\frac{\partial y}{\partial t} = 0$ at $t = 0$.
5. Solve : $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions, $u(0, t) = u(\pi, t) = 0, u(x, 0) = 2\sin 3x - 4\sin 5x$.
6. Solve $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 5u$, when $u(x, 0) = 5e^{-3x} + 7e^{2x}$
7. Consider the conduction of heat along a bar which is covered by a material impervious to heat under the condition :
   i) $V \neq \infty$ as $t \to \infty$
   ii) $\frac{\partial v}{\partial x} = 0$ when $x = 0$ and $x = l$
   iii) $V = lx - x^2$ when $t = 0$ between $x = 0$ and $x = l$. then solve $\frac{\partial v}{\partial t} = k\frac{\partial^2 v}{\partial x^2}$
8. Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions :
   (i) $u = 0$ when $y \to \infty$
   (ii) $u = 0$ when $x = 0$
   (iii) $u = 0$ when $x = 1$
   (iv) $u = x(1 - x)$ when $y = 0$ for $0 < x < 1$.
9. Solution of $\frac{\partial^2 z}{\partial x\,\partial y} = \sec x \cos y$ is z……..

10. Solution of $\frac{\partial^2 z}{\partial x\,\partial t} = e^{2t-3x}$ is z……..

11. Solution of $\frac{\partial^2 z}{\partial x\,\partial y} = x^2 \cos y$ is z……..

12. A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by
$y(x,0) = y_0 \sin^3(\frac{\pi x}{L})$ if it is released from rest position, find the displacement y at any distance x from one end and at any time t.

13. Solution of $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0$ is $u = \cdots$

14. Solution of $\frac{\partial^2 u}{\partial x\,\partial y} = \operatorname{cosec} x \sec y$ is u……..

15. Solution of $\frac{\partial^2 z}{\partial x\,\partial y} = \cot x \cot y$ is z……..

16. If $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends , find the solution with conditions :
$y(0,t) = y(L,t) = 0, \frac{\partial y}{\partial t} = 0$ at $t = 0$. $y(x,0) = y_0 \sin\frac{\pi x}{2}$.

17. Solve : $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions, $u(0,t) = u(\pi,t) = 0, u(x,0) = 2\sin 4x - 3\sin 2x$

18. Solution of $\frac{\partial^2 u}{\partial x\,\partial y} = e^{-x} \cos y$ is u……..

19. Solution of $\frac{\partial^2 u}{\partial x\,\partial t} = x^2 e^{2t}$ is u……..

20. Solution of $\frac{\partial^2 u}{\partial x\,\partial y} = x^2 y^2$ is u……..

21. Solve $\frac{\partial^2 z}{\partial x^2} = 4 \frac{\partial^2 y}{\partial y^2}$ satisfying the conditions $z = \frac{\partial z}{\partial y} = \sin x$ when $y = 0$.

22. A rod of length L with insulated sides is initially at uniform temperature u_0, its ends are suddenly cooled to 0^0 C and are kept at that temperature. Find the temperature function $u(x,t)$.

23. Solution of $\frac{\partial^2 u}{\partial x\,\partial t} = e^{-t} \cos x$ is u……..

24. Solution of $\frac{\partial^2 z}{\partial x\,\partial y} = \sin x \sin y$ is z……..

25. Solution of $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$ is $u = \cdots$

26. Solve $\frac{\partial^2 z}{\partial x^2} = 4 \frac{\partial^2 y}{\partial y^2}$ satisfying the conditions $z = \frac{\partial z}{\partial y} = \cos x$ when $y = 0$.

27. The one dimensional wave equation is represented by partial differential equation ……

28. The Solution of $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ is $u = \cdots$

29. The Solution of $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ is $u = \cdots$

30. Solve : : $\frac{\partial u}{\partial t} = a^2 \ \frac{\partial^2 u}{\partial x^2}$ where $u(x,t)$ satisfies the following conditions :

(i)$u0,t) = 0$

(ii) $\frac{\partial u}{\partial x}_{(l,t)} = 0$ for all t

(iii) $u(x,0) = 0, 0 < x < l$

(iv) $u(x,\infty)$ is finite.

31. If $\frac{\partial^2 y}{\partial t^2} = c^2 \ \frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends , find the solution with conditions :

(i)$y(0,t) = 0$

(ii) $y(l,t) = 0$

(iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

(iv) $y(x,0) = k(lx - x^2), 0 < x < l.$

32. Solve: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by method of separation of variables.

33. Solve the boundary value problem: $\frac{\partial^2 y}{\partial t^2} = 4 \ \frac{\partial^2 y}{\partial x^2}$, $y(0,t) = y(5,t) = 0, y(x,0) = 0$ and $\frac{\partial y}{\partial t} = 5 \sin\pi x$ when $t = 0$. [S.E.Oct/Nov 2011]

34. Solve the boundary value problem: $\frac{\partial u}{\partial t} = a^2 \ \frac{\partial^2 u}{\partial x^2}$ subject to the conditions:
$\frac{\partial u}{\partial x}(0,t) = 0, \frac{\partial u}{\partial t}(5,t) = 0, u(x,0) = 5x - x^2, 0 < x < 5$ and u is not infinite for $x \to \infty$.
[S.E.Oct/Nov 2011]

35. Solve: $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u$, $u(x,0) = 3e^{-5x} + 2e^{-3x}$ [S.E.Oct/Nov 2011]

36. Solve the boundary value problem: $\frac{\partial^2 y}{\partial t^2} = c^2 \ \frac{\partial^2 y}{\partial x^2}$

Subject to the conditions:

(i) $y(0,t) = 0$

(ii) $y(l,t) = 0$

(iii) $\frac{\partial y}{\partial t} = 0$, when $t = 0$

(iv) $y(x,0) = \frac{x}{100}(l^2 - x^2)$ for $0 \le x \le l$ [S.E.Oct/Nov 2012]

37. Solve: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin\left(\frac{n\pi x}{l}\right)$ [S.E.Oct/Nov 2011]

38. Solve the boundary value problem: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1, t > 0$

Subject to the conditions:

(i) $u(0,t) = 0$

(ii) $u(1,t) = 0$

(iii) $u(x,0) = 3 \sin m\pi x$ [S.E.Oct/Nov 2012]

39. Solve: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ within the rectangle $0\le x\le a$, $0\le y\le b$ given that
$u(0,y)=0, u(a,y)=0, u(x,b)=0$ and $u(x,0)=x(a-x)$ [S.E.Oct/Nov 2012]
40. Solve: $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+3u$ subject to the conditions $u=0, \frac{\partial u}{\partial x}=2+4e^{-4y}$ when $x=0$ [S.E.Oct/Nov 2012]
41. A string is stretched tightly between $x=0$ and $x=l$ and both the ends are given the displacement $y=\sin 2t$ perpendicular to the string. If the string satisfies the differential $\frac{\partial^2 y}{\partial t^2}=4\frac{\partial^2 y}{\partial x^2}$, find the oscillation of the string. [S.E.May/June 2012]
42. Solve: $2\frac{\partial u}{\partial x}-3\frac{\partial u}{\partial y}=0$ when $u(0,y)=3e^{-y}$ [S.E.May/June 2012]
43. Obtain the solution of $\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}$ subject to the condition
(i) $u(0,t)=u(\pi,t)=0$
(ii) $u(x,0)=x$ in every $(0,\pi)$ [S.E.May/June 2012]
44. Solve: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(x,0)=3\sin n\pi x, u(0,t)=0$ for $0<x<1$ [B.tech. May/June 2013]
45. Solve: $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ subject to the conditions $y(0,t)=0, y(l,t)=0, \frac{\partial y}{\partial t}=0$ at $t=0$ and $y(x,0)=40x-x^2$,$0\le x\le 40$ [B.tech. May/June 2013]
46. solve: $2x\frac{\partial z}{\partial x}-3y\frac{\partial z}{\partial y}=0$ [B.tech. May/June 2013]

Unit-V Z- transform

1. If $Z[f(n)]=f(z)$ then : $Z[a^n f(n)]=$………. [B.Tech May/June 2012]
2. For $n\ge 0, Z[e^{-2n}\cos 3n]=$………… [B.Tech May/June 2012]
3. For $n\ge 0, Z[e^{3n}\sinh n]=$………. [B.Tech May/June 2012]
4. Solve: $y_{n+2}-7y_{n+1}+6y_n=3^n$, $y(0)=0, y(1)=3$ [B.Tech May/June 2012]
5. Find: $z\left[\frac{5^n+7^n}{n}\right]$, $n\ge 1$ [B.Tech May/June 2012]
6. For $n\ge 0, Z[n^2]=$ ……… [B.Tech(old) May/June 2012]
7. $z[e^{-an} f(n)]=$………. [B.Tech(old) May/June 2012]
8. For $n\ge 0, Z[3^n e^{3n}]=$……….. [B.Tech(old) May/June 2012]
9. Solve: $f(n+2)+6f(n+1)+9f(n)=2^n$, with $f(0)=0$, $f(1)=0$ using Z-transform.. [B.Tech(old) May/June 2012]
10. Solve: $y_{n+1}-y_n=3n, y(0)=0$ [B.Tech(old) May/June 2012]
11. For $k\ge 0, Z[\sin(\alpha+k)]=$……….. [S.E. Oct Nov 2008]
12. For $k\ge 0, Z[k\,3^k]=$……….. [S.E. Oct Nov 2008]
13. For $k\ge 0, Z[a^k \sin 3k]=$……….. [S.E. Oct Nov 2008]
14. Solve: $f(k+2)+4f(k+1)+3f(k)=0$, $k\ge 0$ if $f(0)=0$, $f(1)=1$

[S.E. Oct Nov 2008]

15. Solve: $12\, f(k+2) - 7f(k+1) + f(k) = 0, k \geq 0.$ Given that $f(0) = 0, f(1) = 3$
[S.E. Oct Nov 2008]

16. For $k \geq 0, Z[\, e^{-3k} \sin 2k] =$……….. [B.Tech Nov/Dec 2011]

17. For $k \geq 0, Z[\, e^{k} \sin kw] =$……….. [B.Tech Nov/Dec 2011]

18. For $k \geq 0, Z[\, 2^{k} \sinh 3k] =$……….. [B.Tech Nov/Dec 2011]

19. Solve: $y_{k+1} + \frac{1}{3}\, y_{k} = \left(\frac{1}{3}\right)^{k}$, $k \geq 0$, if $y(0) = 0.$ [B.Tech Nov/Dec 2011]

20. Solve: $f(k+2) - 2\cos\alpha\, f(k+1) - f(k) = 0$ if $f(0) = 0, f(1) = 1$

21. For $k \geq 0, Z[\, e^{3k} \sin 2k] =$……….. [B.Tech Nov/Dec 2011]

22. For $n \geq 0, Z[\, 2^{n}\ e^{-3n}] =$……….. [B.Tech(old) Nov/Dec 2011]

23. For $n \geq 0, Z[\, n\, e^{an}] =$……….. [B.Tech(old) Nov/Dec 2011]

24. Solve: $f(n+1) - 2\, f(n) = 2^{n},\ n \geq 0, f(0) = 0$ [B.Tech(old) Nov/Dec 2011]

25. If Z[f(n)]=f(z) ,$n \geq 0$ then $Z[e^{an}\, f(n)] =$ ……… [B.Tech Nov/Dec 2009]

26. For $k \geq 0, Z\left[\frac{f(k)}{k}\right] =$……… [B.Tech Nov/Dec 2009]

27. For $k \geq 0, Z[k] =$………. [B.Tech Nov/Dec 2009]

28. Solve: $y_{k+1} + \frac{1}{2}\, y_{k} = \left(\frac{1}{2}\right)^{k}$, $k \geq 0$ if $y(0) = 0$ [B.Tech Nov/Dec 2009]

29. If $Z(u_{n}) = U(z)$ then $Z(a^{-n}\, u_{n}) =$………. [B.Tech May/June 2009]

30. $Z(n^{p})$=…………,P being positive integer [B.Tech May/June 2009]

31. $Z(n^{3}) =$………. [B.Tech May/June 2009]

32. Solve: $y_{n+2} + 6\, y_{n+1} + 9\, y_{n} = 2^{n}$, with $y_0 = y_1 = 0$ using $z -$ transform.
[B.Tech S.E. May/June 2009]

33. Solve : $f(n+1) - f(n) = 0, f(0) = 1$ [B.Tech May/June 2009]

34. Find: $Z[2^{n} \sin\left(\frac{n\pi}{2} + \theta\right)]$ [B.Tech Nov/Dec 2007]

35. Solve $y_{n} - 2y_{n-1} + y_{n-2} = n$ [B.Tech Nov/Dec 2007]

36. Find: $Z\left[\frac{\sin(ak)}{k}\right],\ k \geq 0$

37. Find: $Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}\right],\ |z| > \frac{1}{4}$ [B.Tech Nov/Dec 2007]

38. Find: $Z^{-1}\left[\frac{z^3}{(z-1)\left(z-\frac{1}{2}\right)^{2}}\right]$ [B.Tech Nov/Dec 2007]

39. Find $Z\left[\left(\frac{1}{3}\right)^{n} \sin\left(\frac{n\pi}{2}\right)\right]$ [B.Tech Nov/Dec 2013]

40. Find: $Z[\sin^{3} n]$ [B.Tech Nov/Dec 2013]

41. Find: $Z^{-1}\left[\frac{z}{(z-1)(z-3)}\right]$ [B.Tech Nov/Dec 2013]

42. Find: $Z[3^{n}], n \geq 1$ [B.Tech Nov/Dec 2013]

43. Find: $Z^{-1}\left[\frac{z+1}{z^2-2z+1}\right]$ [B.Tech Nov/Dec 2013]

44. Solve by Z-transform: $f(n) + \frac{1}{4} f(n-1) = u(n) + \frac{1}{3}\, u(n-1)$ [B.TechNov/Dec 2013]

45. Find: $Z\left[\left(\frac{1}{4}\right)^{|n|}\right]$ [B.Tech Nov/Dec 2013]

46. Find Z-transform of $3^k, k < 0$ [S.E. May June 2013]

47. Find Z-transform of $\sin 3k, k \geq 0$ [S.E. May June 2013]

48. Solve: $y(k+2) - 5y(k+1) + 6y(k) = 4^k, y(0) = 0, y(1) = 1$ by Z-transform.

49. Find inverse Z-transform of : $\frac{z}{z^2-2z+2}$ [S.E. May June 2013]

50. Find Z-transform of $e^{-3k}\cos 4k$ [S.E. May June 2013]

51. Find Z-transform of $e^{-3k}\sinh(2k+5)$ [S.E.(Old) May June 2013]

52. Find inverse Z-transform of : $\frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)}$ [S.E.(Old) May June 2013]

53. Solve the equation: $12y_{k+2} - 7y_{k+1} + y_k = 0, k \geq 0, y(0) = 0, y(1) = 3$ [S.E.(Old) May June 2013]

54. Find Z-transform of $2^k\cos(3k+2)$ [S.E. Oct Nov 2011]

55. Find inverse Z-transform of : $\frac{z^2}{(z^2+1)}$ [S.E. Oct Nov 2011]

56. Solve: $y_{k+2} - 5y_{k+1} + 6y_k = k, y(0) = 0, y(1) = 0$ [S.E. Oct Nov 2011]

57. Find inverse Z-transform of : $\frac{10z}{(z-1)(z-2)}, |z| > 2$ [S.E. Nov/Dec 2012]

58. Find Z-transform of $3^k\cosh ak$ [S.E. Nov/Dec 2012]

59. Solve by Z-transform $y_{k+1} + \frac{1}{2}y_k = \left(\frac{1}{2}\right)^k, k \geq 0$ if $y(0) = 0$ [S.E. Nov/Dec 2012]

60. Find: $Z\left[2^n\sin\left(\frac{n\pi}{2} + \theta\right)\right]$ [S.E. May June 2012]

61. Find: $Z^{-1}\left[\frac{2z(z^2-1)}{(z^2+1)^2}\right]$ [S.E. May June 2012]

62. Solve: $y_n - 2y_{n-1} + y_{n-2} = n$ [S.E. May June 2012]

63. Find inverse Z-transform of : $\frac{z^3}{\left(z-\frac{1}{4}\right)(z-1)}$ [B.Tech Nov/Dec 2013]

64. Using Z-transform Solve: $u_{k+2} - 2u_{k+1} + u_k = 2^k, y_0 = 2, y_1 = 1$ [B.Tech Nov/Dec 2013]

65. If $Z[f(n)] = \bar{f}(z)$ then $Z\left[\frac{f(n)}{n}\right] =$……….. [B.Tech Nov/Dec 2013]

66. If $Z[f(k)] = \bar{f}(z)$ then $Z[k^n a^k] =$ ………. [B.Tech Nov/Dec 2013]

67. Find Z-transform of $\sin\left(\frac{k\pi}{2} + \alpha\right), k \geq 0$ [B.Tech May/June 2013]

68. Find: $Z\left[\frac{2^k+3^k}{k}\right], k \geq 1$ [B.Tech Nov/Dec 2013]

69. Find inverse Z-transform of : $\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}, |z| > \frac{1}{2}$ [B.Tech Nov/Dec 2013]

70. Solve: $f(n+2) + 3f(n+1) + 2f(n) = 0$ with $y(0) = 0, f(1) = 1$

71. Find: $Z[2^k\sinh\alpha k]$ [S.E. Nov/Dec 2009]

72. Solve: $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0, u_1 = 1$ [S.E. Nov/Dec 2009]

73. Find: $\frac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}$, $|z| > \frac{1}{4}$ [S.E. May/June 2009]

74. Find $Z[4^k \sin(2k+3)], k \geq 0$ [S.E. May/June 2009]

Unit-VI Numerical Method

1. Solve the system 6x+y+z=105, 4x+8y+3z=155, 5x+4y-10z=65 using Gauss-Seidal method.
2. Find y(0.2) by Picard's method ,given that : $\frac{dy}{dx} = 1 + y^2$,and $y(0) = 0$
3. Using Newton-Raphson method, find real root of $xe^x - 2 = 0$.
4. Find y (1.2) by Runge-kutta method of fourth order of $\frac{dy}{dx} = \sqrt{x^2 + y^2}$ with y(1)=1.5 ,take h=0.1.
5. Find a root of equation $x^3 - 3x + 1 = 0$ to three decimal places by Newton-Raphson method.
6. Using Runge-kutta method of fourth order , find y when x=0.2 given that : $\frac{dy}{dx} = x + y,\ y(0) = 1$ taking $h = 0.1$
7. Use Picard's method to find y at x=0.3 up to third approximation given that : $\frac{dy}{dx} = x - y$ with $y(0) = 1$
8. Apply Gauss-elimination method to solve the equations: $2x + 3y + z = 13, x - y - 2z = -1,\ 3x + y + 4z = 15$
9. Find a root of equation $x^3 - 4x + 1 = 0$ to three decimal places by Newton-Raphson method.
10. Use Picard's method to obtain y for x=1.1 for the differential equation: $(1 + y^2)dy = x^2dx$
    With y(0)=0.
11. Apply Gauss-Seidal method to solve the equations: $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$
12. Evaluate $\sqrt{28}$ to four decimal places by Newton –Raphson method.
13. Solve $\frac{dy}{dx} = \frac{1}{x+y}$ for x=0.5 to x=1, h=0.5 by using Runge-kutta fourth order method with y (0) =1.
14. Find the value of y for x=0.1 by Picard's method given that : $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.
15. Apply Gauss-seidal iteration method to solve the equations: $2x + 3y + z = 9\ , x + 2y + 3z = 6\ , 3x + y + 2z = 8$
16. Using Runge-kutta method of order 4 , to find y when x=0.4, given that $\frac{dy}{dx} = x + y^2$, y(0)=1,h=0.2.

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