

Unit-1 Function of complex variable

1. If $f(z)$ is analytic in a closed curve c , except at a finite number of singular points within c , then: $\int_c f(z) dz = \dots\dots$ [B.tech.Nov/Dec 2012]
2. The value of $\int_0^{\pi i} \cos z dz = \dots\dots$ [B.tech.Nov/Dec 2012]
3. If $f(z)$ has a pole of order three at $z = a$ then: $\text{Res}f(a) = \dots$ [B.tech.Nov/Dec 2012]
4. If C is the circle $|z|= 2$ then $\int_c \frac{1}{z-1} dz = \dots\dots$ [B.tech.Nov/Dec 2012]
5. Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2+y^2}$ are harmonic functions of (x,y) but not harmonic conjugates. [B.tech.Nov/Dec 2012]
6. Evaluate: $\int_c \frac{2z^2+z}{z^2-1} dz$ where c is $|z - 1| = 1$ by Cauchy integral formula. [B.tech.Nov/Dec 2012]
7. Evaluate $\int_c |z|^2 dz$ around the square with vertices at $(0,0), (1,0), (1,1)$ and $(0,1)$. [B.tech.Nov/Dec 2012]
8. Find the image of the circle $|z|= 2$ under the transformation $w = z + 3 + 2i$. [B.tech.Nov/Dec 2012]
9. If $f(z)$ is analytic within and on a closed curve and if 'a' is any point within c , then $\frac{1}{2\pi i} \int_c \frac{f(z)}{z-a} dz = \dots\dots$ [B.tech. May/June 2012]
10. The value of $\int_0^{2i} \sinh z dz = \dots\dots$ [B.tech. May/June 2012]
11. The poles of the function $f(z) = \frac{z^3-1}{z^3+1}$ are $\dots\dots$ [B.tech. May/June 2012]
12. If C is the circle $|z|= 2$ then $\int_c \frac{e^z}{(z-3)^2} dz = \dots\dots$ [B.tech. May/June 2012]
13. Let $f(z) = u + iv$ be an analytic function, if $u = 3x - 2xy$ then find v and express $f(z)$ in terms of z . [B.tech. May/June 2012]
14. Evaluate: $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 2$ by Cauchy integral formula. [B.tech. May/June 2012]
15. Find the image of the lines :
 i) $x = y + 1$
 ii) The line joining $A (1+i)$ to $B (2+3i)$ in z -plane under the transformation $W = \frac{i}{z}$
 [B.tech. May/June 2012]
16. Evaluate: $\int_0^{\infty} \frac{dx}{(1+x^2)^2}$ using residue theorem.
17. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function at $z = x + iy$, find $f(z)$ in terms of z .
18. Evaluate: $\int_c \frac{z}{(z-1)(z-3)} dz$ where c is the circle $|z| = 3$ [B.tech. May/June 2011]

19. If $f(z)$ is analytic at all points within C in a multiple connected region R then,
 $\int_c f(z)dz = \dots\dots$ [B.tech. May/June 2012]
20. If C is the circle $|z| = \frac{1}{2}$ then $\int_c \frac{e^{-z}}{z+1} dz = \dots\dots$ [B.tech. May/June 2011]
21. If C is the circle $|z| = 1$ then $\int_c \frac{e^{-z}}{z^2} dz = \dots\dots$ [B.tech. May/June 2011]
22. Evaluate: $\int_c \frac{3z^2+z}{z^2-1} dz$ where c is $|z - 1| = 1$ [B.tech. May/June 2011]
23. Cauchy-Riemann equation in Cartesian coordinates are.....
 [B.tech. Oct/Nov 2011]
24. If $f(z)$ is analytic function and $f'(z)$ is continuous at each point within and on a closed curve C , then $\int_c f(z)dz = \dots\dots$ [B.tech. Oct/Nov 2011]
25. If $f'(z) = \cos x \cosh y - i \sin x \sinh y$ then $f(z) = \dots\dots$
 [B.tech. Oct/Nov 2011]
26. If C is the circle $|z - 1| = 1$ then $\int_c \frac{\cos z}{z} dz = \dots\dots$ [B.tech. Oct/Nov 2011]
27. Find p such that the function: $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic.
 [B.tech. Oct/Nov 2011]
28. Find the analytic function: $f(z) = u + iv$ given $r = a(1 + \cos\theta)$
29. Evaluate: $\int_c \frac{e^z}{\cos \pi z} dz$ where C is the circle $|z| = 1$ [B.tech. May/June 2010]
30. If $f(z)$ is analytic within and on a closed curve C then $\int_c \frac{f(z)}{z-a} dz = \dots\dots$
 [B.tech. May/June 2010]
31. If $f(z)$ is a single valued and analytic within and on a closed curve C then
 $\int_c \frac{f(z)}{(z-a)^{n+1}} dz = \dots\dots$ [B.tech. May/June 2010]
32. Every analytic function $f(z) = u(x, y) + iv(x, y)$ defines two families of curves
 $u(x, y) = C_1$ and $v(x, y) = C_2$. These family of curves are.....to each other.
 [B.tech. May/June 2010]
33. Prove that the function $\cosh z$ is analytic and find its derivative.
 [B.tech. May/June 2010]
34. If C is the circle $|z| = 2$ then evaluate $\int_c \frac{e^{-z}}{z+1} dz$ [B.tech. May/June 2010]
35. The necessary and sufficient condition for the function
 $w = f(z) = u(x, y) + iv(x, y)$ to be analytic in a region R , are.....
 [B.tech. May/June 2010]
36. An electrostatic field in the xy plane is given by the potential function :
 $\phi = 3x^2y - y^3$. Find the stream function. [B.tech. May/June 2011]
37. Evaluate: $\int_c \frac{2z-1}{z(z+1)(z-3)} dz$ where c is the circle $|z| = 2$ [B.tech. May/June 2010]
38. If C is the circle $|z - 1| = 3$ then $\int_c \frac{\cos z}{z-\pi} dz = \dots\dots$ [B.tech. May/June 2010]
39. Determine p such that the function $f(z) = \frac{1}{2} \text{Log}(x^2 + y^2) + i \tan^{-1} \frac{px}{y}$
 is analytic function. [B.tech.(Old) May/June 2010]

40. Determine analytic function $w = u + iv$ if $v = \log(x^2 + y^2) + x - 2y$
[B.tech.(Old) May/June 2010]
41. Cauchy-Riemann equation in polar coordinates are..... [B.tech. Oct/Nov 2009]
42. C-R equation in Cartesian form = [B.tech. Oct/Nov 2009]
43. Residue of $e^{\frac{1}{z^2}} \cos z = \dots\dots$ [B.tech. Oct/Nov 2009]
44. Residue of $\frac{\tan z}{z}$ at $z = 0$ is..... [B.tech. Oct/Nov 2009]
45. If $f(z)$ has a pole of order m at $z = a$ then the residue of $f(z)$ at $z = a$ is.....
[B.tech. Oct/Nov 2009]
46. Prove that an analytic function with constant amplitude is constant.
[B.tech. Oct/Nov 2009]
47. Evaluate: $\int_C \frac{e^{3z}}{z - \pi i} dz$ where C is the ellipse $|z - 2| + |z + 2| = 6$
[B.tech. Oct/Nov 2009]
48. If $f(z) = u + iv$ analytic function, find $f(z)$ if $u + v = r^2(\cos 2\theta + \sin 2\theta)$ when $f(0) = 0$. [B.tech. Oct/Nov 2009]
49. Evaluate using residue theorem $\int_C \frac{z-3}{z^2+2z+5} dz$ where $C: |z + 1 - i| = 3$.
[B.tech. Oct/Nov 2009]
50. Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle $|z - 2i| = \frac{3}{2}$
[B.tech. May/June 2009]
51. If $u + v = e^x(\cos y + \sin y) + \frac{x-y}{x^2+y^2}$, find the analytic function $f(z) = u + iv$
Where $f(1) = 1$. [B.tech. May/June 2009]
52. Evaluate: $\int_0^{2\pi} \frac{d\theta}{(5-3\cos \theta)^2}$ [B.tech. May/June 2009]
53. Find Laurent's expansion for $f(z) = \frac{1}{(1-z^2)(z+2)}$
For i) $|z| < 1$ ii) $1 < |z| < 2$ [B.tech. May/June 2009]
54. Find an analytic function $f(z)$ such that
 $\operatorname{Re}[f'(z)] = 3x^2 - 4y - 3y^2$ and $f(1 + i) = 0$ [S.E. Oct/Nov 2011]
55. Evaluate: $\int_1^i \frac{\operatorname{Log}^3 z}{z} dz$ along the arc of the circle $|z| = 1$ [S.E. Oct/Nov 2011]
56. Prove that $u = r^2 \cos 2\phi$ is harmonic function and find its harmonic conjugate and also corresponding analytic function. [S.E. Oct/Nov 2011]
57. Evaluate $\int_C \frac{\sin z}{(z-1)^2(z^2-9)} dz$ where C is the circle :
(i) $|z - 3i| = 1$ (ii) $|z - 3| = \frac{1}{2}$ [S.E. Oct/Nov 2011]
58. If $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ find Laurent's series expansion in
(i) $0 < |z - 1| < 4$ (ii) $|z - 1| > 4$ [S.E. Oct/Nov 2011]
59. Evaluate by Cauchy's residue theorem $\int_C \frac{1}{z \sin z} dz$ where C is the circle $|z| = \frac{1}{2}$

60. Find the image of the square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$ under the transformation $W = (1 + i)z + (2 + i)$ [S.E.Oct/Nov 2011]
61. Evaluate: $\int_0^{2\pi} \frac{2}{2+\cos\theta} d\theta$ [S.E.Oct/Nov 2011]
62. Find the bilinear transformation which maps the points $1, -1, \infty$ in z -plane on to the points $1, i, -1$ in w -Plane. [S.E.Oct/Nov 2011]
63. Evaluate: $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$ ($a > 0$) [S.E.Oct/Nov 2011]
64. Determine whether the function $f(z) = z e^{-z}$ is analytic or not. [S.E.Nov/Dec 2012]
65. Evaluate: $\int_0^{\pi-\pi i} e^{\bar{z}} dz$ along the curve $C: x = t, y = -t$ [S.E.Nov/Dec 2012]
66. Prove that: $v = r^2 \sin 2\phi - \frac{1}{r} \sin \phi$ is harmonic and find its harmonic conjugates and also corresponding analytic function. [S.E.Nov/Dec 2012]
67. Evaluate: $\int_C \frac{z^2}{z^4-1} dz$ where C is the circle
 (i) $|z| = \frac{1}{2}$ (ii) $|z - 1| = \frac{1}{2}$ [S.E.Nov/Dec 2012]
68. Expand $\cos z$ in to a Taylor's series about the point $z = \frac{\pi}{2}$ [S.E.Nov/Dec 2012]
69. Evaluate by Cauchy's residue theorem: $\int_C \frac{z}{\sin^2 z} dz$ where $C: |z| = \frac{1}{5}$ [S.E.Nov/Dec 2012]
70. Find the bilinear transformation which maps the points $2, i, -2$ in z -plane on to the points $1, i, -1$ in w -Plane. [S.E.Nov/Dec 2012]
71. Evaluate: $\int_{-\infty}^\infty \frac{dx}{(x^2+1)^2}$ [S.E.Nov/Dec 2012]
72. Find the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$ [S.E.May/June 2012]
 Evaluate: $\int_C \operatorname{Re}(z) dz$ where C is the semi unit circle. [S.E.May/June 2012]
73. Evaluate by using Cauchy's integral formula: $\int_C \frac{z+1}{z^2+2z+4} dz$ where C is $|z + 1 + i| = 2$ [S.E.May/June 2012]
74. Evaluate: $\oint_C \frac{\coth z}{z-i} dz$ where C is $|z| = 2$ by Cauchy's residue theorem. [S.E.May/June 2012]
75. Expand $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ for $1 < |z + 1| < 3$ [S.E.May/June 2012]
76. Find the bilinear transformation which maps the points $z = -1, 2, i - 1$ in to the points $W = \infty, \frac{2i}{3}, i - 1$. [S.E.May/June 2012]
77. Find the analytic function whose real part is $y + e^x \cos y$ [S.E.Nov/Dec 2008]
78. Evaluate: $\int_C (z - z^2) dz$ where C is upper half of the circle $|z| = 1$ [S.E.Nov/Dec 2008]
79. Apply Cauchy's residue theorem to evaluate: $\oint_C \frac{dz}{(z^2+1)(z^2+4)}$ where C is the circle $|z| = 1.5$ [S.E.Nov/Dec 2008]

80. Determine the bilinear transformation that maps the points: $1 - 2i, 2 + i, 2 + 3i$ respectively into $2 + 2i, 1 + 3i, 4$. [S.E.Nov/Dec 2008]
81. Use Cauchy's integral formula to evaluate: $\int_C \frac{z^2 - 2z + 1}{(z-i)^2} dz$ where C is $|z| = 2$ [S.E.Nov/Dec 2008]
82. Prove that $u = \text{Log}(x^2 + y^2)$ is harmonic function and find its harmonic conjugate and also corresponding analytic function. [S.E.Nov/Dec 2008]
83. If the real part of an analytic function $f(z)$ is $x^2 - y^2 - y$ then find imaginary part. [S.E.Nov/Dec 2014]
84. If $f(z) = \frac{a}{2} r \cos\theta + i(r \sin\theta + 2)$ is harmonic, then find the value of a. [S.E.Nov/Dec 2014]
85. Find the imaginary part of analytic function whose real part is $r^{-4} \cos 4\theta$ [S.E.Nov/Dec 2014]
Find whether the function $u = \text{Log}|z|^2$ is harmonic. If so find analytic function whose real part is u. [S.E.Nov/Dec 2014]
86. Show that the image of the line $x = 0$ under the transformation $w = e^z$ is a circle. Evaluate $\int_0^{1+i} (x + iy) dz$ along $y = x$ [S.E.Nov/Dec 2014]
Evaluate $\int_C (z - i) dz$ where C is 0 to i [S.E.Nov/Dec 2014]
87. Find the poles of $f(z)$ and residues at the poles which lie on imaginary axis if $f(z) = \frac{z^2 - 2z}{(z+1)(z^2+4)}$ [S.E.Nov/Dec 2014]
88. Find the Bilinear transformation which maps the points $z = 1, -i, 1$ onto the points $w = 2, i, -2$ [S.E.Nov/Dec 2014]
89. Evaluate $\int_{(0,0)}^{(1,1)} (3x^2 + 4xy + ix^2) dx$ along $y = x^2$ [S.E.Nov/Dec 2014]
90. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ for $0 < |z - 1| < 1$ [S.E.Nov/Dec 2014]
91. Evaluate $\oint_C \frac{dz}{\sinh^2 z}$ where C is $|z| = 2$ by using Cauchy's integral formula.
92. Under the transformation $W = z + \frac{a^2 - b^2}{4z}$ maps the circle of radius $\frac{1}{2}(a + b)$ with centre at origin in the z-plane into an ellipse on W-plane.
93. Evaluate $\oint_C \frac{\cosh z}{(z+1)^3(z-1)} dz$ where C is $|z| = 2$ by Cauchy's Residue theorem.
94. Evaluate: $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$ using residue theorem. [S.E.Nov/Dec 2014]

Unit-II Application of Complex Variable

1. Evaluate $\int_C |z|^2 dz$ around the square with vertices at (0,0),(1,0),(1,1) and (0,1).
[B.tech.Nov/Dec 2012]
2. Find the image of the lines :
 - i) $x = y + 1$
 - ii) The line joining A (1+i) to B (2+3i) in z-plane under the transformation $W = \frac{i}{z}$
3. Evaluate: $\int_1^i \frac{\text{Log } z^3}{z} dz$ along the arc of the circle $|z| = 1$
4. Find the image of the square with vertices (0,0) , (2,0) (2,2) (0,2) under the transformation $W = (1 + i)z + (2 + i)$ [S.E.Oct/Nov 2011]
5. Evaluate: $\int_0^{\pi - \pi i} e^{\bar{z}} dz$ along the curve C: $x = t, y = -t$ [S.E.Nov/Dec 2012]
6. Find the bilinear transformation which maps the points $2, i, -2$ in z-plane on to the points $1, i, -1$ in w-Plane. [S.E.Nov/Dec 2012]
7. Find the bilinear transformation which maps the points $1, -1, \infty$ in z-plane on to
8. Find the bilinear transformation which maps the points $z = -1, 2, i - 1$ in to the points $W = \infty, \frac{2i}{3}, i - 1$
9. Determine the bilinear transformation that maps the points: $1 - 2i, 2 + i, 2 + 3i$ respectively into $2 + 2i, 1 + 3i, 4$.
10. Evaluate $\int_0^{1+i} (x + iy) dz$ along $y = x$
11. Evaluate $\int_C (z - i) dz$ where C is 0 to i
12. Find the Bilinear transformation which maps the points $z = 1, -i, 1$ onto the points $w = 2, i, -2$
13. Evaluate $\int_{(0,0)}^{(1,1)} (3x^2 + 4xy + ix^2) dx$ along $y = x^2$
14. Under the transformation $W = z + \frac{a^2 - b^2}{4z}$ maps the circle of radius $\frac{1}{2}(a + b)$ with centre at origin in the z-plane into an ellipse on W-plane

Unit-III Vector Integration

1. Find the work done by the force $\vec{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ along the curve between $(1, -2, 1)$ to $(3, 1, 4)$
2. Find the work done by the force $\vec{F} = (2xz^3 + 6y)\mathbf{i} + (6x - 2yz)\mathbf{j} + (3x^2z^2 - y^2)\mathbf{k}$ along the curve from $(0, 0, 0)$ to $(1, 0, 0)$ then $(1, 0, 0)$ to $(1, 1, 1)$
3. Find the work done by the force $\vec{F} = 2xyz^2\mathbf{i} + [x^2z^2 + z \cos(yz)]\mathbf{j} + [2x^2yz + y \cos(yz)]\mathbf{k}$ from $(1, 0, 1)$ to $(0, \frac{\pi}{2}, 1)$
4. If $\vec{F} = x^2\mathbf{i} - xy\mathbf{j}$ then find $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r}$ along the parabola $y = x^2$
5. Find the work done when a force $\vec{F} = x^2\mathbf{i} + xy\mathbf{j}$ moves a particle in x-y plane from $(0, 0)$ to $(1, 1)$ along the path $y^2 = x$
6. Calculate the work done in moving a particle in force field given by $\vec{F} = x^2y^2\mathbf{i} + y\mathbf{j}$ along $x^2 = 4y$ from $(0, 0)$ to $(4, 4)$
7. If a force $\vec{F} = 2x^2y\mathbf{i} + 3xy\mathbf{j}$ displaces a particle in the xy-plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$ find the work done.
8. Find the work done by a force $\vec{F} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ along $x = a \cos t, y = a \sin t, z = at$ from $t = 0$ to $t = \frac{\pi}{4}$
9. Find the work done in moving a particle once round an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ in the plane $z = 0$ in the force field given by: $\vec{F} = (3x - 2y)\mathbf{i} + (2x + 3y)\mathbf{j} + y^2\mathbf{k}$
10. Apply Green's theorem to evaluate $\int_C [e^{-x} \sin y dx + e^{-x} \cos y dy]$ where C is the rectangle whose vertices are $(0, 0), (\pi, 0), (\pi, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$
11. Apply Green's theorem to evaluate: $\int_C (xy + y^2) dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$

12. Apply Green's theorem to evaluate: $\int_C \bar{F} \, d\bar{r}$ where $\bar{F} = x^2\mathbf{i} + xy\mathbf{j}$ and C is a triangle having vertices A (0, 2), B (2, 0) and C (4, 2)

13. Apply Green's theorem to evaluate $\int_C \bar{F} \, d\bar{r}$ where $\bar{F} = (y - \sin x)\mathbf{i} + \cos x \mathbf{j}$ over

$$y = 0, x = \frac{\pi}{2}, y = \frac{2x}{\pi}$$

14. Apply Green's theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ for

$$\bar{F} = (x^2 - 2xy)\mathbf{i} + (xy^2 + 3)\mathbf{j}, y^2 = 8x, x = 2$$

15. Using Green's theorem evaluate $\int_C \bar{F} \, d\bar{r}$ where

$$\bar{F} = (2x^2y + 3z^2)\mathbf{i} + (x^2 + 4yz)\mathbf{j} + (2y^2 + 6xz)\mathbf{k} \text{ and } C \text{ is the curve enclosing a region } y^2 = 4ax, x = a \text{ in the plane } z = 0$$

16. Apply Green's theorem to evaluate: $\int_C \bar{F} \, d\bar{r}$ where $\bar{F} = 2y^2\mathbf{i} + 3x\mathbf{j}$ over $y = x^2$ and

$$y = x$$

17. Apply Stoke's theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$, $\bar{F} = y^2\mathbf{i} + xy\mathbf{j} + xz\mathbf{k}$ where C is the

boundary of the curve of hemisphere $x^2 + y^2 + z^2 = a^2$, $z > 0$ in positive direction.

18. Apply Stoke's theorem to evaluate $\int (x^2 + y - 4)dx + 3xydy + (2xz + z^2)dz$

Over the surface of the hemisphere $x^2 + y^2 + z^2 = 9$

19. Apply Stokes-theorem evaluate $\int_C [(x + 2y)dx + (x - z)dy + (y - z)dz]$ where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6).

20. Apply stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ for

$$\bar{F} = 2y(1 - x)\mathbf{i} + (x - x^2 - y^2)\mathbf{j} + (x^2 + y^2 + z^2)\mathbf{k} \text{ over the area of the triangle}$$

$$x + y + z = 2$$

21. Evaluate: $\iint_S (\nabla \times \bar{F}) \cdot ds$ where $\bar{F} = (x^3 - y^3)\mathbf{i} - xyz\mathbf{j} + y^3\mathbf{k}$ and S is the surface

$$x^2 + 4y^2 + z^2 - 2x = 4 \text{ above the plane } x = 0$$

22. Evaluate: $\iint_S \bar{F} \cdot \hat{n} \, ds$ as a volume integral if $\bar{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ over $x^2 + y^2 + z^2 = a^2$

23. Evaluate $\iint_S (x^3 dydz + x^2 ydzdx + x^2 zdx dy)$ where S surface bounded by the cylinder $z = 0, z = 3$ & $x^2 + y^2 = 4$

24. Evaluate: $\iint_S (x^3 i + y^3 j + z^3 k) ds$ where $x^2 + y^2 + z^2 = 16$.

25. Evaluate: $\iint_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = 4xi - 2y^2j + z^2k$ and S is the region bounded by $y^2 = 4x, x = 1, z = 0, z = 3$

26. Evaluate: $\iint_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = (2xy + z)i + y^2j - (x + 3y)k$ over the volume bounded by $2x + 2y + z = 6, x = 0, y = 0, z = 0$.

Unit-IV Application of partial differential equation

- The solution of $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial y}$ is $u = \dots\dots$
- The solution of $\frac{\partial^2 z}{\partial x \partial y} = \cot x \cos y$ is $z = \dots\dots$
- The solution of : $\frac{\partial u}{\partial x} = 3 \frac{\partial u}{\partial t} + u$ is $u = \dots\dots$
- If $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L, fixed at both ends, find the solution with conditions :
 $y(0, t) = y(L, t) = 0, y(x, 0) = y_0 \sin \frac{\pi x}{L}$ and $\frac{\partial y}{\partial t} = 0$ at $t = 0$.
- Solve : $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions, $u(0, t) = u(\pi, t) = 0, u(x, 0) = 2 \sin 3x - 4 \sin 5x$.
- Solve $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 5u$, when $u(x, 0) = 5e^{-3x} + 7e^{2x}$
- Consider the conduction of heat along a bar which is covered by a material impervious to heat under the condition :
 i) $V \neq \infty$ as $t \rightarrow \infty$
 ii) $\frac{\partial v}{\partial x} = 0$ when $x = 0$ and $x = l$
 iii) $V = lx - x^2$ when $t = 0$ between $x = 0$ and $x = l$. then solve $\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2}$
- Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions :
 (i) $u = 0$ when $y \rightarrow \infty$
 (ii) $u = 0$ when $x = 0$
 (iii) $u = 0$ when $x = 1$
 (iv) $u = x(1 - x)$ when $y = 0$ for $0 < x < 1$.
- Solution of $\frac{\partial^2 z}{\partial x \partial y} = \sec x \cos y$ is $z = \dots\dots$

10. Solution of $\frac{\partial^2 z}{\partial x \partial t} = e^{2t-3x}$ is z
11. Solution of $\frac{\partial^2 z}{\partial x \partial y} = x^2 \cos y$ is z
12. A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{L}\right)$ if it is released from rest position, find the displacement y at any distance x from one end and at any time t .
13. Solution of $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0$ is $u = \dots$
14. Solution of $\frac{\partial^2 u}{\partial x \partial y} = \operatorname{cosec} x \operatorname{sec} y$ is u
15. Solution of $\frac{\partial^2 z}{\partial x \partial y} = \cot x \cot y$ is z
16. If $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends, find the solution with conditions :
 $y(0, t) = y(L, t) = 0, \frac{\partial y}{\partial t} = 0$ at $t = 0$. $y(x, 0) = y_0 \sin \frac{\pi x}{2}$.
17. Solve : $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions, $u(0, t) = u(\pi, t) = 0, u(x, 0) = 2\sin 4x - 3\sin 2x$
18. Solution of $\frac{\partial^2 u}{\partial x \partial y} = e^{-x} \cos y$ is u
19. Solution of $\frac{\partial^2 u}{\partial x \partial t} = x^2 e^{2t}$ is u
20. Solution of $\frac{\partial^2 u}{\partial x \partial y} = x^2 y^2$ is u
21. Solve $\frac{\partial^2 z}{\partial x^2} = 4 \frac{\partial^2 y}{\partial y^2}$ satisfying the conditions $z = \frac{\partial z}{\partial y} = \sin x$ when $y = 0$.
22. A rod of length L with insulated sides is initially at uniform temperature u_0 , its ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature function $u(x, t)$.
23. Solution of $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ is u
24. Solution of $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ is z
25. Solution of $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$ is $u = \dots$
26. Solve $\frac{\partial^2 z}{\partial x^2} = 4 \frac{\partial^2 y}{\partial y^2}$ satisfying the conditions $z = \frac{\partial z}{\partial y} = \cos x$ when $y = 0$.
27. The one dimensional wave equation is represented by partial differential equation
28. The Solution of $3 \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$ is $u = \dots$
29. The Solution of $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ is $u = \dots$

30. Solve : $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ where $u(x, t)$ satisfies the following conditions :
- $u(0, t) = 0$
 - $\frac{\partial u}{\partial x}(l, t) = 0$ for all t
 - $u(x, 0) = 0, 0 < x < l$
 - $u(x, \infty)$ is finite.
31. If $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends, find the solution with conditions :
- $y(0, t) = 0$
 - $y(l, t) = 0$
 - $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$
 - $y(x, 0) = k(lx - x^2), 0 < x < l.$
32. Solve: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by method of separation of variables.
33. Solve the boundary value problem: $\frac{\partial^2 y}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2}, y(0, t) = y(5, t) = 0, y(x, 0) = 0$ and $\frac{\partial y}{\partial t} = 5 \sin \pi x$ when $t = 0$. [S.E.Oct/Nov 2011]
34. Solve the boundary value problem: $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions:
 $\frac{\partial u}{\partial x}(0, t) = 0, \frac{\partial u}{\partial t}(5, t) = 0, u(x, 0) = 5x - x^2, 0 < x < 5$ and u is not infinite for $x \rightarrow \infty$.
 [S.E.Oct/Nov 2011]
35. Solve: $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u, u(x, 0) = 3e^{-5x} + 2e^{-3x}$ [S.E.Oct/Nov 2011]
36. Solve the boundary value problem: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$
 Subject to the conditions:
 - $y(0, t) = 0$
 - $y(l, t) = 0$
 - $\frac{\partial y}{\partial t} = 0$, when $t = 0$
 - $y(x, 0) = \frac{x}{100}(l^2 - x^2)$ for $0 \leq x \leq l$ [S.E.Oct/Nov 2012]
37. Solve: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$ [S.E.Oct/Nov 2011]
38. Solve the boundary value problem: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$
 Subject to the conditions:
 - $u(0, t) = 0$
 - $u(1, t) = 0$
 - $u(x, 0) = 3 \sin m\pi x$ [S.E.Oct/Nov 2012]

39. Solve: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ within the rectangle $0 \leq x \leq a, 0 \leq y \leq b$ given that $u(0, y) = 0, u(a, y) = 0, u(x, b) = 0$ and $u(x, 0) = x(a - x)$ [S.E.Oct/Nov 2012]
40. Solve: $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 3u$ subject to the conditions $u = 0, \frac{\partial u}{\partial x} = 2 + 4e^{-4y}$ when $x = 0$ [S.E.Oct/Nov 2012]
41. A string is stretched tightly between $x = 0$ and $x = 1$ and both the ends are given the displacement $y = \sin 2t$ perpendicular to the string. If the string satisfies the differential $\frac{\partial^2 y}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2}$, find the oscillation of the string. [S.E.May/June 2012]
42. Solve: $2 \frac{\partial u}{\partial x} - 3 \frac{\partial u}{\partial y} = 0$ when $u(0, y) = 3e^{-y}$ [S.E.May/June 2012]
43. Obtain the solution of $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to the condition
 (i) $u(0, t) = u(\pi, t) = 0$
 (ii) $u(x, 0) = x$ in every $(0, \pi)$ [S.E.May/June 2012]
44. Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(x, 0) = 3 \sin n\pi x, u(0, t) = 0$ for $0 < x < 1$ [B.tech. May/June 2013]
45. Solve: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ subject to the conditions $y(0, t) = 0, y(l, t) = 0, \frac{\partial y}{\partial t} = 0$ at $t = 0$ and $y(x, 0) = 40x - x^2, 0 \leq x \leq 40$ [B.tech. May/June 2013]
46. solve: $2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$ [B.tech. May/June 2013]

Unit-V Z- transform

1. If $Z[f(n)] = f(z)$ then : $Z[a^n f(n)] = \dots\dots\dots$ [B.Tech May/June 2012]
2. For $n \geq 0, Z[e^{-2n} \cos 3n] = \dots\dots\dots$ [B.Tech May/June 2012]
3. For $n \geq 0, Z[e^{3n} \sinh n] = \dots\dots\dots$ [B.Tech May/June 2012]
4. Solve: $y_{n+2} - 7y_{n+1} + 6y_n = 3^n, y(0) = 0, y(1) = 3$ [B.Tech May/June 2012]
5. Find: $z \left[\frac{5^n + 7^n}{n} \right], n \geq 1$ [B.Tech May/June 2012]
6. For $n \geq 0, Z[n^2] = \dots\dots\dots$ [B.Tech(old) May/June 2012]
7. $z[e^{-an} f(n)] = \dots\dots\dots$ [B.Tech(old) May/June 2012]
8. For $n \geq 0, Z[3^n e^{3n}] = \dots\dots\dots$ [B.Tech(old) May/June 2012]
9. Solve: $f(n+2) + 6f(n+1) + 9f(n) = 2^n$, with $f(0) = 0, f(1) = 0$ using Z-transform.. [B.Tech(old) May/June 2012]
10. Solve: $y_{n+1} - y_n = 3n, y(0) = 0$ [B.Tech(old) May/June 2012]
11. For $k \geq 0, Z[\sin(\alpha + k)] = \dots\dots\dots$ [S.E. Oct Nov 2008]
12. For $k \geq 0, Z[k 3^k] = \dots\dots\dots$ [S.E. Oct Nov 2008]
13. For $k \geq 0, Z[a^k \sin 3k] = \dots\dots\dots$ [S.E. Oct Nov 2008]
14. Solve: $f(k+2) + 4f(k+1) + 3f(k) = 0, k \geq 0$ if $f(0) = 0, f(1) = 1$

- [S.E. Oct Nov 2008]
15. Solve: $12f(k+2) - 7f(k+1) + f(k) = 0, k \geq 0$. Given that $f(0) = 0, f(1) = 3$
[S.E. Oct Nov 2008]
16. For $k \geq 0, Z[e^{-3k} \sin 2k] = \dots\dots\dots$ [B.Tech Nov/Dec 2011]
17. For $k \geq 0, Z[e^k \sin kw] = \dots\dots\dots$ [B.Tech Nov/Dec 2011]
18. For $k \geq 0, Z[2^k \sinh 3k] = \dots\dots\dots$ [B.Tech Nov/Dec 2011]
19. Solve: $y_{k+1} + \frac{1}{3} y_k = \left(\frac{1}{3}\right)^k, k \geq 0$, if $y(0) = 0$. [B.Tech Nov/Dec 2011]
20. Solve: $f(k+2) - 2 \cos \alpha f(k+1) - f(k) = 0$ if $f(0) = 0, f(1) = 1$
21. For $k \geq 0, Z[e^{3k} \sin 2k] = \dots\dots\dots$ [B.Tech Nov/Dec 2011]
22. For $n \geq 0, Z[2^n e^{-3n}] = \dots\dots\dots$ [B.Tech(old) Nov/Dec 2011]
23. For $n \geq 0, Z[n e^{an}] = \dots\dots\dots$ [B.Tech(old) Nov/Dec 2011]
24. Solve: $f(n+1) - 2f(n) = 2^n, n \geq 0, f(0) = 0$ [B.Tech(old) Nov/Dec 2011]
25. If $Z[f(n)] = f(z), n \geq 0$ then $Z[e^{an} f(n)] = \dots\dots\dots$ [B.Tech Nov/Dec 2009]
26. For $k \geq 0, Z\left[\frac{f(k)}{k}\right] = \dots\dots\dots$ [B.Tech Nov/Dec 2009]
27. For $k \geq 0, Z[k] = \dots\dots\dots$ [B.Tech Nov/Dec 2009]
28. Solve: $y_{k+1} + \frac{1}{2} y_k = \left(\frac{1}{2}\right)^k, k \geq 0$ if $y(0) = 0$ [B.Tech Nov/Dec 2009]
29. If $Z(u_n) = U(z)$ then $Z(a^{-n} u_n) = \dots\dots\dots$ [B.Tech May/June 2009]
30. $Z(n^p) = \dots\dots\dots, P$ being positive integer [B.Tech May/June 2009]
31. $Z(n^3) = \dots\dots\dots$ [B.Tech May/June 2009]
32. Solve: $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, with $y_0 = y_1 = 0$ using z -transform.
[B.Tech S.E. May/June 2009]
33. Solve : $f(n+1) - f(n) = 0, f(0) = 1$ [B.Tech May/June 2009]
34. Find: $Z\left[2^n \sin\left(\frac{n\pi}{2} + \theta\right)\right]$ [B.Tech Nov/Dec 2007]
35. Solve $y_n - 2y_{n-1} + y_{n-2} = n$ [B.Tech Nov/Dec 2007]
36. Find: $Z\left[\frac{\sin(ak)}{k}\right], k \geq 0$
37. Find: $Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}\right], |z| > \frac{1}{4}$ [B.Tech Nov/Dec 2007]
38. Find: $Z^{-1}\left[\frac{z^3}{(z-1)\left(z-\frac{1}{2}\right)^2}\right]$ [B.Tech Nov/Dec 2007]
39. Find $Z\left[\left(\frac{1}{3}\right)^n \sin\left(\frac{n\pi}{2}\right)\right]$ [B.Tech Nov/Dec 2013]
40. Find: $Z[\sin^3 n]$ [B.Tech Nov/Dec 2013]
41. Find: $Z^{-1}\left[\frac{z}{(z-1)(z-3)}\right]$ [B.Tech Nov/Dec 2013]
42. Find: $Z[3^n], n \geq 1$ [B.Tech Nov/Dec 2013]
43. Find: $Z^{-1}\left[\frac{z+1}{z^2-2z+1}\right]$ [B.Tech Nov/Dec 2013]
44. Solve by Z-transform: $f(n) + \frac{1}{4}f(n-1) = u(n) + \frac{1}{3}u(n-1)$ [B.TechNov/Dec 2013]

45. Find: $Z \left[\left(\frac{1}{4} \right)^{|n|} \right]$ [B.Tech Nov/Dec 2013]
46. Find Z-transform of $3^k, k < 0$ [S.E. May June 2013]
47. Find Z-transform of $\sin 3k, k \geq 0$ [S.E. May June 2013]
48. Solve: $y(k+2) - 5y(k+1) + 6y(k) = 4^k, y(0) = 0, y(1) = 1$ by Z-transform.
49. Find inverse Z-transform of: $\frac{z}{z^2 - 2z + 2}$ [S.E. May June 2013]
50. Find Z-transform of $e^{-3k} \cos 4k$ [S.E. May June 2013]
51. Find Z-transform of $e^{-3k} \sinh(2k + 5)$ [S.E.(Old) May June 2013]
52. Find inverse Z-transform of: $\frac{z^2}{(z-1)(z-\frac{1}{2})}$ [S.E.(Old) May June 2013]
53. Solve the equation: $12y_{k+2} - 7y_{k+1} + y_k = 0, k \geq 0, y(0) = 0, y(1) = 3$ [S.E.(Old) May June 2013]
54. Find Z-transform of $2^k \cos(3k + 2)$ [S.E. Oct Nov 2011]
55. Find inverse Z-transform of: $\frac{z^2}{(z^2 + 1)}$ [S.E. Oct Nov 2011]
56. Solve: $y_{k+2} - 5y_{k+1} + 6y_k = k, y(0) = 0, y(1) = 0$ [S.E. Oct Nov 2011]
57. Find inverse Z-transform of: $\frac{10z}{(z-1)(z-2)}, |z| > 2$ [S.E. Nov/Dec 2012]
58. Find Z-transform of $3^k \cosh k$ [S.E. Nov/Dec 2012]
59. Solve by Z-transform $y_{k+1} + \frac{1}{2}y_k = \left(\frac{1}{2}\right)^k, k \geq 0$ if $y(0) = 0$ [S.E. Nov/Dec 2012]
60. Find: $Z \left[2^n \sin \left(\frac{n\pi}{2} + \theta \right) \right]$ [S.E. May June 2012]
61. Find: $Z^{-1} \left[\frac{2z(z^2 - 1)}{(z^2 + 1)^2} \right]$ [S.E. May June 2012]
62. Solve: $y_n - 2y_{n-1} + y_{n-2} = n$ [S.E. May June 2012]
63. Find inverse Z-transform of: $\frac{z^3}{(z - \frac{1}{4})(z-1)}$ [B.Tech Nov/Dec 2013]
64. Using Z-transform Solve: $u_{k+2} - 2u_{k+1} + u_k = 2^k, y_0 = 2, y_1 = 1$
[B.Tech Nov/Dec 2013]
65. If $Z[f(n)] = \bar{f}(z)$ then $Z \left[\frac{f(n)}{n} \right] = \dots\dots\dots$ [B.Tech Nov/Dec 2013]
66. If $Z[f(k)] = \bar{f}(z)$ then $Z[k^n a^k] = \dots\dots\dots$ [B.Tech Nov/Dec 2013]
67. Find Z-transform of $\sin \left(\frac{k\pi}{2} + \alpha \right), k \geq 0$ [B.Tech May/June 2013]
68. Find: $Z \left[\frac{2^k + 3^k}{k} \right], k \geq 1$ [B.Tech Nov/Dec 2013]
69. Find inverse Z-transform of: $\frac{z^2}{(z - \frac{1}{2})(z - \frac{1}{3})}, |z| > \frac{1}{2}$ [B.Tech Nov/Dec 2013]
70. Solve: $f(n+2) + 3f(n+1) + 2f(n) = 0$ with $y(0) = 0, f(1) = 1$
71. Find: $Z[2^k \sinh ak]$ [S.E. Nov/Dec 2009]
72. Solve: $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0, u_1 = 1$ [S.E. Nov/Dec 2009]

73. Find: $\frac{z^2}{(z-\frac{1}{4})(z-\frac{1}{5})}$, $|z| > \frac{1}{4}$

[S.E. May/June 2009]

74. Find $Z[4^k \sin(2k + 3)]$, $k \geq 0$

[S.E. May/June 2009]

Unit-VI Numerical Method

1. Solve the system $6x+y+z=105$, $4x+8y+3z=155$, $5x+4y-10z=65$ using Gauss-Seidal method.
2. Find $y(0.2)$ by Picard's method ,given that : $\frac{dy}{dx} = 1 + y^2$, and $y(0) = 0$
3. Using Newton-Raphson method, find real root of $xe^x - 2 = 0$.
4. Find $y(1.2)$ by Runge-kutta method of fourth order of $\frac{dy}{dx} = \sqrt{x^2 + y^2}$ with $y(1)=1.5$,take $h=0.1$.
5. Find a root of equation $x^3 - 3x + 1 = 0$ to three decimal places by Newton-Raphson method.
6. Using Runge-kutta method of fourth order , find y when $x=0.2$ given that :
 $\frac{dy}{dx} = x + y$, $y(0) = 1$ taking $h = 0.1$
7. Use Picard's method to find y at $x=0.3$ up to third approximation given that :
 $\frac{dy}{dx} = x - y$ with $y(0) = 1$
8. Apply Gauss-elimination method to solve the equations:
 $2x + 3y + z = 13$, $x - y - 2z = -1$, $3x + y + 4z = 15$
9. Find a root of equation $x^3 - 4x + 1 = 0$ to three decimal places by Newton-Raphson method.
10. Use Picard's method to obtain y for $x=1.1$ for the differential equation: $(1 + y^2)dy = x^2 dx$
With $y(0)=0$.
11. Apply Gauss-Seidal method to solve the equations:
 $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$
12. Evaluate $\sqrt{28}$ to four decimal places by Newton -Raphson method.
13. Solve $\frac{dy}{dx} = \frac{1}{x+y}$ for $x=0.5$ to $x=1$, $h=0.5$ by using Runge-kutta fourth order method with $y(0)=1$.
14. Find the value of y for $x=0.1$ by Picard's method given that :
 $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.
15. Apply Gauss-seidal iteration method to solve the equations:
 $2x + 3y + z = 9$, $x + 2y + 3z = 6$, $3x + y + 2z = 8$
16. Using Runge-kutta method of order 4 , to find y when $x=0.4$, given that $\frac{dy}{dx} = x + y^2$,
 $y(0)=1, h=0.2$.

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