Unit-I Differential Equations

1. Solve : $\frac{(2xy+1)}{y}\,dx + \frac{y-x}{y^2}\,dy = 0$
2. Solve: $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$
3. Find the orthogonal trajectories of the family of curve: $r = a(1-\cos\theta)$
4. Solve: $\frac{dy}{dx} - xy = y^2 e^{-x^2/2}\log x$
5. A resistance of 100 ohms, an inductance of 0.5 Henry is connected in series with a battery of 20 volts. Find the current in a circuit as a function of t.
6. A particle falls under gravity in a resisting medium of which the resistance varies as the velocity. If the particle starts from rest, find the velocity at any time t.
7. Solve : $\left(1+2e^{\frac{x}{y}}\right)dx + 2\,e^{\frac{x}{y}}\left(1-\frac{x}{y}\right) = 0$
8. Solve: $\text{Sin}2x\,\frac{dy}{dx} = y + \tan x$
9. Find the orthogonal trajectories of : $\left(r+\frac{k^2}{r}\right)\cos\theta = \alpha$
10. Solve : $\sin y\,\frac{dy}{dx} = (1 - x\cos y)\cos y$
11. The equation of the electromotive force in terms of current i for an electrical circuit having resistance R , and a condenser of capacity C in series is : $E = Ri + \int \frac{i}{C}\,dt$ $E =$ Find the current i, when $E = E_m\sin\omega t$.
12. A particle of mass m is projected vertically upwards under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is: $\frac{v^2}{g}[\lambda - \log(1+\lambda)]$ where v is the greatest velocity which above mass will attain when it falls freely and λv is the initial velocity.
13. Solve: $\left[\cos x\,\text{Log}_e(2y-8) + \frac{1}{x}\right]dx + \frac{\sin x}{y-4}\,dy = 0$ with $y(1) = \frac{9}{2}$
14. Solve: $x(x-1)\frac{dy}{dx} - (x-2)y = x^2(2x-1)$
15. Find the orthogonal trajectories of : $r^n = a^n\cos n\theta$
16. An e.m.f. is connected in series with resistance R an inductance L, where L=640,R=250,E=500.
    i)Form the differential equation for the circuit.
    ii)show that current will approaches 2 amps as t increases.
    iii)Find in how many seconds i will approach 90% of its maximum value.
17. A body of mass m falling from rest is subjected to the force of gravity and air resistance of k times of (velocity) ^{2}.If it falls through a distance x and possesses a velocity v at that instant, prove that : $\frac{2kx}{m} = \log\left(\frac{a^2}{a^2-v^2}\right)$ where $mg=ka^2$
18. Solve: $(1+\sin y)\frac{dx}{dy} = 2y\cos y - x(\sec y + \tan y)$

19. Solve : $(2xy^4 + \sin y)dx + (4x^2y^3 + x\cos y)dy = 0$
20. Find the orthogonal trajectories of the family of curve: $r = \frac{2a}{1+\cos\theta}$
21. Solve: $(1 + y^2) + \left(x - e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$
22. Solve:$3x(1 - x^2)y^2\frac{dy}{dx} + (2x^2 - 1)y^3 = ax^3$
23. Solve: $\frac{dy}{dx} + 2y\tan x = y^2\tan^2 x$
24. A particle of mass m under gravity in a medium whose resistance is k times velocity where k is constant. If the particle is projected vertically upwards with velocity V,show that the time to reach the highest point is :
    $\frac{m}{k}\ \log[1 + \frac{KV}{mg}]$
25. Under what condition the equation: $(\cosh y + \cos x)dx + bx\sinh y\, dy = 0$ is exact?
26. Find the integrating factor of : $\sec^2 y\ \frac{dy}{dx} + x\tan y = x^3$
27. $\frac{dy}{dx} - y\tan x = y^4\sec x$
28. Solve: $(x + 1)\frac{dy}{dx} - y = e^{3x}\ (x + 1)^2$
29. Solve: $\frac{dy}{dx} + x\sin 2y = x^3\ \cos^2 y$
30. Solve: $r\sin\theta - \frac{dr}{d\theta}\ \cos\theta = r^2$
31. A particle falls under gravity in a resisting medium whose resistance varies as the velocity. If the particle starts from rest, find the velocity at any time t.
32. Given $L\frac{di}{dt} + Ri = E$:
    i)Find current i
    ii)show that current will approaches 2 amps as t increases.(when L=540,R=150,E=300)
    iii) Find in how many seconds i will approach 90% of its maximum value
33. Solve: $e^{-y}\sec^2 y\, dy = dx + x\, dy$
34. Solve: $\frac{dy}{dx} = -\left(\frac{x+y\cos x}{1+\sin x}\right),\ \ y\left(\frac{\pi}{2}\right) = 1$
35. Find the orthogonal trajectories of the family of curve: $r^m = a^m\ \cos m\theta$
36. Solve: $y^2\frac{dx}{dy} + xy = 2y^2 + 1$
37. Solve: $\frac{dy}{dx} = \frac{y(y-e^x)}{e^x-2xy}$
38. Find the orthogonal trajectories of the family of curve:$ay^2 = x^3$
39. A particle is projected vertically upwards with velocity V_1 and resistance of the air produces retardation KV^2, where V is the velocity. Find the greatest height attained by the particle.
40. Solve: $y\, e^x dx = (y^3 + 2xe^y)dy$
41. Solve: $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$
42. Solve: $(1 + x^2)\ \frac{dy}{dx} + y = e^{\tan^{-1}x}$

43. In an electric circuit containing resistance R ,an inductance L,the voltage E and current i Are connected by the equation : $E = Ri + L\frac{di}{dt}$, If L=320,R=150,E=450 and i=0 when t=0.show that the current i will approach 3 amp as t increases.
44. A moving body is opposed by a force per unit mass of value cx and resistance per unit mass of value $bv^{2,}$ where x and v are the displacement and velocity of the particle at that instant.Find the velocity of the particle in terms of x,if it starts from rest.
45. Define exact differential equation.
46. Find the integrating factor of : $(1 + y^2)dx = (\tan^{-1}y - x)dy$
47. Solve:$3y^2 \frac{dy}{dx} + 2y^3x = 4xe^{-x^2}$
48. Solve: $(1 + 2xy\cos x^2 - 2xy)dx + (\sin x^2 - x^2)dy = 0.$
49. Solve: $y\log y\, dx + (x - \log y)dy = 0$
50. Define linear differential equation and Bernoulli's differential equation.
51. Solve: $y\frac{dx}{dy} - x = 2y^3$ [F.E.Nov/Dec 2013]
52. Solve: $\frac{dy}{dx} + y\tan x = y^3\sec x$ [F.E.Nov/Dec 2013]
53. Solve: $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ [F.E.Nov/Dec 2013]
54. Find the orthogonal trajectories of the parabola: $y^2 = 4ax$ [F.E.Nov/Dec 2013]
55. Find the orthogonal trajectories of the curve: $xy = c$ [F.E.Nov/Dec 2013]
56. Find the solution of exact differential equation:
    $(x + 2y - 2)dx + (2x - y + 3)dy = 0$ [F.E.April/May 2012]
57. Solve: $(\sec x\tan x\tan y - e^x)dx + (\sec x\sec^2 y)dy = 0$ [F.E.April/May 2012]
58. Solve: $\tan y\frac{dy}{dx} - \cos y\cos^2 x = -\tan x$ [F.E.April/May 2012]
59. Solve the equation: $L\frac{di}{dt} + Ri = 20\cos(3t)$ where $R = 10$ ohms, $L = 0.5$ henry. Given that $i = 0$ when $t = 0$. [F.E.April/May 2012]
60. Solve: $\frac{dx}{dy} = \frac{2xy}{x^2 - y^2}$ [F.E.Nov/Dec 2008]
61. Solve: $\sin x\frac{dx}{dt} - \cos x + t\cos^2 x = 0$ [F.E.Nov/Dec 2008]
62. Solve: $\cos^2 x\frac{dy}{dx} - \tan x = -y$ [F.E.Nov/Dec 2008]
63. A condenser of capacity c is charged through a resistance R by steady voltage V, show that the charge q on the plate is given by : $R\frac{dq}{dt} + \frac{q}{c} = v$ hence show that if $q = 0$ at $t = 0$, $q = cv\left[1 - e^{-\frac{t}{RC}}\right]$ [F.E.Nov/Dec 2008]
64. Solve: $(x + a)\frac{dy}{dx} - 3y = (x + a)^5$ [F.E.May/June 2008]
65. Solve: $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$ [F.E.May/June 2008]
66. Solve: $\left(y^2e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$ [F.E.May/June 2008]
67. Find the orthogonal trajectories of the family of curve: $x^2 + cy^2 = 1$

[F.E.May/June 2008]

68. A circuit containing resistance of 20 ohms and all inductance 10 henries is connected to 100 volts supply. Determine current after 2 seconds. [F.E.May/June 2008]

69. Show that $\frac{g}{n^2}\log(\cosh nt)$ is the distance passed over by a body falling vertically from rest, assuming that the resistance of air is $\frac{n^2}{g}$ times the square of the velocity. [F.E.May/June 2008]

70. Solve: $(x + \tan y)dy = \sin 2y\, dx$ [F.E.Nov/Dec 2007]

71. Solve: $x\, dx + y\, dy = \frac{a(x\, dy - y\, dx)}{x^2+y^2}$ [F.E.Nov/Dec 2007]

72. Solve: $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$ [F.E.Nov/Dec 2007]

73. Find the orthogonal trajectories of the family of curve: $r = a(1 + \cos\theta)$ [F.E.Nov/Dec 2007]

74. The equation of L-R series circuit is given by $L\frac{di}{dt} + Ri = 4\sin 3t$ if $i = 0$ at $t = 0$ then express i as function of t. [F.E.Nov/Dec 2007]

75. Find the integrating factor of $(1 + y^2)dx = [\tan^{-1}y - x]dy$ [F.E.Nov/Dec 2013]

76. Solve: $[\cos x \tan y + \cos(x + y)]dx + [\sin x \sec^2 x + \cos(x + y)]dy = 0$ [F.E.Nov/Dec 2007]

77. Solve: $y\frac{dx}{dy} - x = 2y^2$ [F.E.Nov/Dec 2013]

78. Find the orthogonal trajectories of the family of curve: $r^2 = c\sin 2\theta$ [F.E.Nov/Dec 2013]

79. Find the integrating factor of : $R\frac{dQ}{dt} + \frac{Q}{C} = V$ [B.Tech Nov/Dec 2013]

80. A constant emf E volts is applied to an electrical circuit containing resistance R and inductance L in series. If the initial current is zero show that the time for current to build up to half of its maximum is: $\frac{L\log 2}{R}$ sec. [B.Tech Nov/Dec 2013]

81. A particle falls in a vertical line under gravity and air resistance to its motion is proportional to its velocity and distance as function of t.shoow that the velocity V will never exceed $\frac{g}{k}$. [B.Tech Nov/Dec 2013]

82. Write a equation for R-C circuit. [B.Tech May/June 2015]

83. $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$ is exact differential equation. Justify your answer.

84. Find I.F. of: $\frac{dy}{dx} + x^2y = x^5$ [B.Tech May/June 2015]

85. Solve: $y^2\frac{dx}{dy} + xy = 2y^2 + 1$ [B.Tech May/June 2015]

86. Solve: $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ [B.Tech May/June 2015]

87. Find the orthogonal trajectory of the family of the curve: $x^2 + y^2 = 2ax$

88. In an electric circuit containing resistance R , an inductance L,the voltage and current i are connected by equation : $L\frac{di}{dt} + Ri = E$. If $L = 540$, $R = 150$, $E = 300$ and $i = 0$ when $t = 0$.Show that current will approach 2 amps as t increases. Also find in how many seconds i will approach 90% of the maximum value.

89. Solve: $r\sin\theta - \frac{dr}{d\theta}\cos\theta = r^2$ [B.Tech May/June 2015]

Unit-II Application of Differential Equations

1. A resistance of 100 ohms, an inductance of 0.5 Henry is connected in series with a battery of 20 volts. Find the current in a circuit as a function of t.
2. A particle falls under gravity in a resisting medium of which the resistance varies as the velocity. If the particle starts from rest, find the velocity at any time t.
3. The equation of the electromotive force in terms of current i for an electrical circuit having resistance R , and a condenser of capacity C in series is : $E = Ri + \int \frac{i}{C}\,dt$ $E =$ Find the current i, when $E = E_m \sin\omega t$.
4. A particle of mass m is projected vertically upwards under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is: $\frac{v^2}{g}[\lambda - \log(1+\lambda)]$ where v is the greatest velocity which above mass will attain when it falls freely and λv is the initial velocity.
5. An e.m.f. is connected in series with resistance R an inductance L, where L=640,R=250,E=500.
   i)Form the differential equation for the circuit.
   ii)show that current will approaches 2 amps as t increases.
   iii)Find in how many seconds i will approach 90% of its maximum value.
6. A body of mass m falling from rest is subjected to the force of gravity and air resistance of k times of (velocity) ^{2}.If it falls through a distance x and possesses a velocity v at that instant, prove that : $\frac{2kx}{m} = \log\left(\frac{a^2}{a^2-v^2}\right)$ where $mg=ka^2$
7. A particle of mass m under gravity in a medium whose resistance is k times velocity where k is constant. If the particle is projected vertically upwards with velocity V,show that the time to reach the highest point is :
   $\frac{m}{k}\log\left[1 + \frac{KV}{mg}\right]$
8. A particle falls under gravity in a resisting medium whose resistance varies as the velocity. If the particle starts from rest, find the velocity at any time t.
9. Given $L\frac{di}{dt} + Ri = E$:
   i)Find current i
   ii)show that current will approaches 2 amps as t increases.(when L=540,R=150,E=300)

iii) Find in how many seconds i will approach 90% of its maximum value

10. A particle is projected vertically upwards with velocity V_1 and resistance of the air produces retardation KV^2, where V is the velocity. Find the greatest height attained by the particle.
11. In an electric circuit containing resistance R ,an inductance L,the voltage E and current i Are connected by the equation : $E = Ri + L\frac{di}{dt}$, If L=320,R=150,E=450 and i=0 when t=0.show that the current i will approach 3 amp as t increases.
12. A moving body is opposed by a force per unit mass of value cx and resistance per unit mass of value bv^{2}, where x and v are the displacement and velocity of the particle at that instant.Find the velocity of the particle in terms of x,if it starts from rest.
13. Solve the equation: $L\frac{di}{dt} + Ri = 20\cos(3t)$ where $R = 10$ ohms, $L = 0.5$ henry. Given that $i = 0$ when $t = 0$. [F.E.April/May 2012]
14. A condenser of capacity c is charged through a resistance R by steady voltage V, show that the charge q on the plate is given by : $R\frac{dq}{dt} + \frac{q}{c} = v$ hence show that if $q = 0$ at $t = 0$, $q = cv\left[1 - e^{-\frac{t}{RC}}\right]$ [F.E.Nov/Dec 2008]
15. A circuit containing resistance of 20 ohms and all inductance 10 henries is connected to 100 volts supply. Determine current after 2 seconds. [F.E.May/June 2008]
16. Show that $\frac{g}{n^2}\log(\cosh nt)$ is the distance passed over by a body falling vertically from rest, assuming that the resistance of air is $\frac{n^2}{g}$ times the square of the velocity.
17. The equation of L-R series circuit is given by $L\frac{di}{dt} + Ri = 4\sin 3t$ if $i = 0$ at $t = 0$ then express i as function of t. [F.E.Nov/Dec 2007]
18. A constant emf E volts is applied to an electrical circuit containing resistance R and inductance L in series. If the initial current is zero show that the time for current to build up to half of its maximum is: $\frac{L\log 2}{R}$ sec. [B.Tech Nov/Dec 2013]
19. A particle falls in a vertical line under gravity and air resistance to its motion is proportional to its velocity and distance as function of t.shoow that the velocity V will never exceed $\frac{g}{k}$. [B.Tech Nov/Dec 2013]
20. In an electric circuit containing resistance R , an inductance L,the voltage and current i are connected by equation : $L\frac{di}{dt} + Ri = E$. If $L = 540$, $R = 150$, $E = 300$ and $i = 0$ when $t = 0$.Show that current will approach 2 amps as t increases. Also find in how many seconds i will approach 90% of the maximum value.

Unit-III Curve Tracing

1. Trace the curve $r = \frac{a}{2}(1 + \cos\theta)$ with full justification.
2. Find the total length of the cycloid $x = a(\theta + \sin\theta), y = a(1 + \cos\theta)$ between two consecutive cusps.
3. Trace the curve with full justification $a^2x^2 = y^3(2a - y)$
4. Trace the curve with full justification, $x = at, y = \frac{a}{t}$
5. Trace the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ with full justification.
6. Trace the curve $3ay^2 = x(x - a)^2$ with full justification.
7. Find the equation of the asymptotes for the curve :
   $x^2(x^2 - 4a^2) = y^2(x^2 - a^2)$
8. Find the equation of tangents at the pole for :
   $r^2 = a^2\cos 2\theta$
9. Find $\tan\phi$ for the curve: $r = a(1 + \cos\theta)$
10. Trace the curve: $y^2(x^2 - 1) = x$ with full justification.
11. Find the total length of the curve : $r = a\sin^3(\frac{\theta}{3})$
12. Trace the curve: $x^{2/3} + y^{2/3} = a^{2/3}$ with full justification.
13. Find the total length of the loop of the curve:
    $x = t^2, y = t(1 - \frac{t^2}{3})$
14. Trace the curve: $y^2 = x^2(\frac{a^2 - x^2}{a^2 + x^2})$ with full justification.
15. Trace the curve $r = 2\sin 2\theta$ with full justification.
16. Trace the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$
17. Trace the curve $x(x^2 + y^2) = a(x^2 - y^2)$ with full justification.
18. Trace the curve: $y^2(2a - x) = x^3$ with full justification.
19. Find the total length of the cycloid $x = a(t + \sin t), y = a(1 + \cos t)$ between two consecutive cusps.
20. Trace the curve with full justification $3y^2 = x(x - 3)^2$
21. Trace the curve $r^2 = \cos 2\theta$ with full justification
22. Trace the curve $y^2(a - x) = x^2(a + x)$ with full justification.
23. Find the length of the arc of the curve $x = e^\theta\left(\sin\frac{\theta}{2} + 2\cos\frac{\theta}{2}\right)$,

    $y = e^\theta\left(\cos\frac{\theta}{2} - 2\sin\frac{\theta}{2}\right)$ from $\theta = 0$ to $\theta = \pi$
24. Trace the curve $x^3 + y^3 = 3axy$ with full justification.
25. Trace the curve $r^2 = a^2\sin 2\theta$ with full justification.
26. Find the equation of the asymptotes for the curve : $x(x^2 + y^2) = a(x^2 - y^2)$, $a > 0$

27. Trace the curve $xy^2 = a(x^2 - a^2)$ with full justification.
28. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ for the curve $x = a(\theta + \sin\theta),\ y = a(1 + \cos\theta)$.
29. Find the length of the curve $y^2 = (2x - 1)^3$ cut-off by the line $x = 4$
30. Find the length of the arc of the curve $y = \mathrm{Log}(\sec x)$ from $x = 0$ to $x = \frac{\pi}{3}$
31. Find the total length of the curve : $r = a(1 + \cos\theta)$
32. Show that the perimeter of the curve $r = a(1 + \cos 2\theta)$ is

$$\frac{2a}{\sqrt{3}}[2\sqrt{3} + \log(2 + \sqrt{3})$$

33. Find the equation of asymptote of the curve: $y^2x^2 = a^2(y^2 - x^2)$ [F.E.Nov/Dec 2013]
34. The curve $x^{1/2} + y^{1/2} = a^{1/2}$ is symmetrical about ………[F.E.Nov/Dec 2013]
35. The length of the curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$ is given by the formula ……… [F.E.Nov/Dec 2013]
36. Trace the curve: $y^2[a^2 + x^2] = x^2[a^2 - x^2]$ with full justification. [F.E.Nov/Dec 2013]
37. Trace the curve $r = 2 + 3\cos\theta$ with full justification. [F.E.Nov/Dec 2013]
38. Find the length of the arc of the curve: $\theta = \frac{1}{2}(r + \frac{1}{r})$ for $r = 1$ to $r = 3$ [F.E.Nov/Dec 2013]
39. Trace the curve $x = a\cos^3 t,\ y = a\sin^3 t$ with full justification. [F.E.Nov/Dec 2013]
40. Find the total length of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ between two consecutive cusps. [F.E.Nov/Dec 2013]
41. Trace the curve: $x = a(\theta - \sin\theta),\ y = a(1 + \cos\theta)$ with full justification. [F.E.Nov/Dec 2009]
42. Trace the curve: $y^2(2a - x) = x^3$ with full justification.[F.E.Nov/Dec 2009]
43. Trace the curve $r = a + b\cos\theta$, $a > b$ with full justification.[F.E.Nov/Dec 2009]
44. Trace the curve $r = a(1 + \cos\theta)$ with full justification.[F.E.Nov/Dec 2009]
45. Find the length of the curve $x = a(\cos\theta + \theta\sin\theta)$,
    $y = a(\sin\theta - \cos\theta)$ from $\theta = 0$ to $\theta = 2\pi$.[F.E.Nov/Dec 2009]
46. Find the length of the loop of the curve: $3ay^2 = x(x - a)^2$.[F.E.Nov/Dec 2009]
47. Equation of asymptotes parallel to x-axis and y-axis is obtained by ………… [F.E.April/May 2012]
    The curve $x = a\cos t, y = a\sin t$ is symmetrical about ……… [F.E.April/May 2012]
48. The curve $r = a(1 + \sin\theta)$ is symmetrical about ……….. [F.E.April/May 2012]
49. The length of arc S of the curve $\theta = f(r)$ from $r = a$ to $r = b$ is given by …….. [F.E.April/May 2012]
50. Trace the curve $r^2 = a^2\cos 2\theta$ with full justification. [F.E.April/May 2012]
51. Find the total length of the curve: $r = a\sin^3\frac{\theta}{3}$ [F.E.April/May 2012]
52. Find the length of the cycloid $x = a(\cos t + t\sin t), y = a(\sin t - \cos t)$ measured from $t = 0$ to $t = \frac{\pi}{2}$ [F.E.April/May 2012]

53. Define cycloid. Trace: $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ with full justification. [F.E.Nov/Dec 2008]
54. Trace the curve: $r = a\cos 3\theta$ with full justification. [F.E.Nov/Dec 2008]
55. Trace the curve: $x^2[x^2 + y^2] = a^2[x^2 - y^2]$ with full justification [F.E.Nov/Dec 2008]
56. Show that in the catenary $y = c\cosh\left(\frac{x}{c}\right)$ the length of the arc from vertex to any point is $s = c\sinh\left(\frac{x}{c}\right)$ [F.E.Nov/Dec 2008]
57. Trace the curve: $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ with full justification. [F.E.Nov/Dec 2008]
58. Trace the curve: $x = a\cos t + \frac{1}{2}a\,\text{Log}\tan^2\left(\frac{t}{2}\right)$, $y = a\sin t$ with full justification [F.E.Nov/Dec 2008]
59. Trace the curve: $(x + a)\,y^2 = x^2(2a - x)$ with full justification [F.E.May/June 2008]
60. Trace the curve: $a\,y^2 = x\,(x - a)^2$ with full justification [F.E.May/June 2008]
61. Trace the curve $r^2 = a^2\sin 2\theta$ with full justification. [F.E.May/June 2008]
62. Find the length of the curve: $x = e^{\theta}\cos\theta$, $y = e^{\theta}\sin\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$ [F.E.May/June 2008]
63. Obtain the total length of the curve: $r^{1/3} = a^{1/3}\sin\left(\frac{\theta}{3}\right)$ [F.E.May/June 2008]
64. Give the procedure for tracing the curve in polar form. [F.E.Nov/Dec 2007]
65. Trace the curve: $r = a\sin 3\theta$ with full justification. [F.E.Nov/Dec 2007]
66. Trace the curve $ay^2 = x^2(x - a)$ with full justification. [F.E.Nov/Dec 2007]
67. Trace the curve $r = 4 + 3\cos\theta$ with full justification. [F.E.Nov/Dec 2007]
68. Find the total length of the curve: $r^2 = 4\cos 2\theta$. [F.E.Nov/Dec 2007]
69. Find the equation of asymptote to the curve: $y^2(a - x) = x^2(a + x)$ [May/June 20015]
70. Curve $r = a + b\cos\theta$, $a < b$ symmetric about …………….[May/June 20015]
71. Trace the curve: $a^4y^2 = x^5(2a - x)$ giving full justification. [May/June 20015]
72. Trace the curve $r = a\sin 3\theta$ with full justification**.** [May/June 20015]
73. Trace the curve $x = a\cos^3 t$, $y = a\sin^3 t$ with full justification.
74. Find the total length of the cycloid $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$
75. Write a formula of length of parametric curve $s =$ …………..

Unit-IV Integral Calculus

1. Evaluate : $\int_0^1 \frac{x}{\sqrt{\log\frac{1}{x}}}\, dx$ [B.tech.(Old) Nov/Dec 2009]
2. Evaluate : $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}\, dx$ [B.tech.(Old) Nov/Dec 2009]
3. Given : $\Gamma 1.8 = 0.9314$, find the value of $\Gamma(-2.2)$ [B.tech. May/June 2010]
4. Evaluate: $\int_0^2 x\,(8-x^3)^{\frac{1}{3}}\, dx$ [B.tech. May/June 2010]
5. Evaluate : $\int_0^\pi \frac{\sin^4\theta}{(1+\cos\theta)^2}\, d\theta$ [B.tech. Nov/Dec 2008]
6. Evaluate: $\int_0^1 \frac{x^{-1/2}}{\sqrt{\log\frac{1}{x}}}\, dx$ [B.tech. Nov/Dec 2008]
7. Evaluate: $\int_0^1 x^3 \sqrt{\frac{1+x^2}{1-x^2}}\, dx$ [B.tech. Nov/Dec 2008]
8. Prove that : $\Gamma m \; \Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}\;\Gamma 2m}{2^{2m-1}}$ [B.tech. May/June 2009]
9. Prove that : $\int_0^\infty \frac{dx}{(e^x+e^{-x})^n} = \frac{1}{4}\,\beta(\frac{n}{2},\frac{n}{2})$ [B.tech. May/June 2009]
10. Prove that : $\int_a^b (x-a)^m (b-x)^n\, dx = (b-a)^{m+n+1}\;\beta(m+1,n+1)$
11. Evaluate: $\int_0^\infty \sqrt{x}\;\; e^{-x^2}\, dx$ [B.tech. Nov/Dec 2010]
12. Evaluate: $\int_0^2 y^4\,(8-y^3)^{1/3}\, dy$ [B.tech. Nov/Dec 2010]
13. Evaluate: $\int_0^1 (\log x)^n\, dx$ [B.tech. Nov/Dec 2012]
14. Evaluate: $\int_0^\infty \frac{x^{10}}{10^x}\, dx$ [B.tech. Nov/Dec 2012]
15. Find the value of : $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x}\; dx$ [B.tech. Nov/Dec 2012]
16. Prove that : $\int_0^\infty \sqrt{y}\; e^{-y^2} dy \times \int_0^\infty \frac{1}{\sqrt{y}}\; e^{-y^2} dy = \frac{\pi}{2\sqrt{2}}$ [B.tech.(Old) Nov/Dec 2012]
17. Prove that : $\int_0^\infty \frac{dx}{(e^x+e^{-x})^n} = \frac{1}{4}\;\beta(\frac{n}{2},\frac{n}{2})$ [B.tech.(Old) Nov/Dec 2012]
18. Evaluate: $\int_0^\infty x^{1/4}\;\; e^{-\sqrt{x}}\, dx$ [B.tech. Nov/Dec 2009]
19. Evaluate: $\int_0^3 \frac{x^3}{\sqrt{1-x}}\, dx$ [B.tech. Nov/Dec 2009]
20. Prove that : $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}\, dx = \frac{1}{a^n b^m}\;\beta(m,n)$ [B.tech.(Old) May/June 2009,2015]
21. Show that : $\int_0^{\pi/2} \sin^m\theta\; \cos^n\theta\, d\theta = \frac{1}{2}\,\frac{\Gamma\frac{m+1}{2}\;\Gamma\frac{n+1}{2}}{\Gamma\left(\frac{m+n}{2}+1\right)}$ [B.tech.(Old) May/June 2009]
22. If $\beta(n,3) = \frac{1}{3}$ n is a positive integer, find n.
23. Evaluate: $\int_0^1 x^3 \log(\frac{1}{x})^4\, dx$
24. Evaluate: $\int_0^\pi x \sin^7 x\;\cos^4 x\;\, dx$

25. Define Gamma function and evaluate: $\int_0^{\infty} e^{-2x}\ x^3\ dx$ [F.E. May/June 2012]

26. Evaluate: $\int_0^{\infty} \frac{x^4}{4^x}\ dx$ [F.E. May/June 2012]

27. Evaluate: $\int_0^{\infty} \frac{x^2}{(1+x^6)^{7/2}}\ dx$ [F.E. May/June 2012]

28. Evaluate: $\int_0^1 x^{n-1} \left(\log\frac{1}{x}\right)^{n-1} dx$ [F.E. May/June 2012]

29. Evaluate: $\int_0^{\pi/2} \sin^{-1/2}\theta\ \cos^{1/2}\theta\ d\theta$ [F.E. May/June 2012]

30. Evaluate: $\int_0^1 \frac{dx}{(1-x^9)^{1/2}}$ [F.E. May/June 2012]

31. Evaluate: $\int_0^1 (x\ \text{Log}x)^4\ dx$ [F.E. May/June 2009]

32. Evaluate: $\int_0^{\infty} \frac{dx}{1+x^4}$ [F.E. May/June 2009]

33. Prove that: $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$ [F.E. May/June 2009]

34. Evaluate: $\int_0^1 x^3(1-x)^4\ dx$ [F.E. Nov/Dec 2012]

35. Solve: $\int_0^{\pi} \sin^3\theta\ \cos^5\theta\ d\theta$ [F.E. Nov/Dec 2012]

36. Evaluate: $\int_0^{\pi/2} \theta \sin^3\theta\ \cos^5\theta\ d\theta$ [F.E. Nov/Dec 2012]

37. Evaluate: $\int_0^1 \sqrt{1-x^4}$ [F.E. Nov/Dec 2012]

38. Evaluate: $\int_0^{\infty} \frac{x^a}{a^x}\ dx$ [F.E. (Old)Nov/Dec 2012]

39. Show that: $\int_0^1 \frac{x^7}{\sqrt{1-x^4}}\ dx = \frac{1}{3}$ [F.E. (Old)Nov/Dec 2012]

40. Prove that: $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\ dx = \beta(m,n)$ [F.E. (Old)Nov/Dec 2012]

41. Solve: $\int_0^{\pi} x \sin^5 x\ \cos^4 x\ dx$ [F.E. (Old)Nov/Dec 2012]

42. Evaluate: $\int_0^{\pi} x\ \cos^6 x\ dx$ [F.E. May/June 2011]

43. Evaluate: $\int_0^{\infty} \frac{x^7(1-x^{12})}{(1+x)^{28}}\ dx$ [F.E. May/June 2011]

44. Evaluate: $\int_0^{\infty} e^{-h^2x^2}\ dx$ [F.E. May/June 2011]

45. Evaluate: $\int_0^{\infty} \frac{x^5}{5^x}\ dx$ [F.E. May/June 2011]

46. Evaluate: $\int_0^{2a} x\sqrt{2ax-x^2}\ dx$ [F.E. Oct/Nov 2011]

47. Evaluate: $\int_0^{2\pi} \text{Sin}^2\theta\ (1+\cos\theta)^4\ d\theta$ [F.E. Oct/Nov 2011]

48. Evaluate: $\int_0^{\infty} x^9\ e^{-2x^2}\ dx$ [F.E. Oct/Nov 2011]

49. Evaluate : $\int_0^1 \frac{dx}{\sqrt{x\ \text{Log}\frac{1}{x}}}$ [F.E. Oct/Nov 2011]

50. Evaluate: $\int_0^{\infty} \sqrt[3]{x^2}\ e^{-\sqrt[3]{x}}\ dx$ [F.E. May/June 2009]

51. Find $\left|\frac{7}{2}\right.$ [B.tech. Nov/Dec 2013]

52. Evaluate: $\int_0^1 x^3\left(1-\sqrt{x}\right)^5\ dx$ [B.tech. Nov/Dec 2013]

53. Evaluate: $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$ [B.tech. Nov/Dec 2013]

54. Prove that : $\int_1^{\infty} \frac{x^{\frac{n}{2}-1}}{(1+x)^{n}} = \frac{1}{2}\, \beta(\frac{n}{2}, \frac{n}{2})$ [B.tech. Nov/Dec 2013]

55. Evaluate: $\int_0^{2\pi} \text{Sin}^4 x \ \cos^6 x\, dx$ [B.tech. May/June 2013]

56. Evaluate: $\int_0^{\infty} x^2\, e^{-x^4}\, dx \ \int_0^{\infty} e^{-x^4}\, dx$ [B.tech. May/June 2013]

57. Evaluate: $\int_0^{1} x^5\, (1-x^5)^{10}\, dx$ [B.tech. May/June 2013]

58. Evaluate: $\int_0^{\infty} \frac{y^8(1-y^6)}{(1+y)^{24}}\, dx$ [B.tech. May/June 2013]

59. Define Beta function. [B.tech. Nov/Dec 2014]

60. Find $\left|\frac{9}{2}\right.$ [B.tech. Nov/Dec 2014]

61. Evaluate: $\int_0^{\infty} \sqrt{x}\ e^{-x^{1/3}}\, dx$ [B.tech. Nov/Dec 2014]

62. Evaluate: $\int_3^{7} \sqrt[4]{(7-x)(x-3)}\, dx$ [B.tech. Nov/Dec 2014]

63. Prove that: $\beta(m,n) \int_0^{\infty} \frac{t^{m-1}}{(1+t)^{m+n}} dt$ [B.tech. Nov/Dec 2014]

64. Evaluate: $\int_0^{\infty} e^{-x}\ x^{2n+1}\, dx$ [F.E. Nov/Dec 2014]

65. Evaluate: $\int_0^{\pi} \sin^6 x\ \cos^4 x\, dx$ [F.E. Nov/Dec 2014]

66. Evaluate: $\beta\left(\frac{1}{2}, \frac{3}{2}\right)$ [F.E. Nov/Dec 2014]

67. Evaluate: $\int_0^{\infty} \sqrt{t}\ e^{-\sqrt{t}}\, dt$ [F.E. Nov/Dec 2014]

68. Evaluate: $\int_0^{1} x^5\, (1-x^3)^{10}\, dx$ [F.E. Nov/Dec 2014]

69. Define Gamma function. [B.tech. May/June 2015]

70. Find: $\int_0^{\pi/4} \cos^3 2t\ \sin^2 4t\, dt$ [B.tech. May/June 2015]

71. Evaluate: $\int_0^{\infty} \sqrt[4]{t}\, e^{-\sqrt{t}}\, dt$ [B.tech. May/June 2015]

72. Evaluate: $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ [B.tech. May/June 2015]

Unit-V Multiple Integrals

1. Evaluate : $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2)dx\, dy$
2. Evaluate : $\int_0^{x/2} \int_0^{a\sin\theta} \int_0^{\frac{a^2-r^2}{a}} r\, dz\, dr\, d\theta$
3. Change the order of integration and evaluate : $\int_0^{1} \int_y^{\sqrt{y}} xy\, dx\, dy$
4. Evaluate: $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} dx\, dy$ by changing to polar coordinates.
5. Evaluate: $\int_0^{1} \int_0^{\sqrt{1-x^2}} \frac{dx\, dy}{1+x^2+y^2}$
6. Evaluate : $\iint_A (x+y)dx\, dy$ where domain A is the area between $y = x^2$ and $y = x$.

7. Change the order of integration by showing the region of integration and evaluate it :
$\int_0^1 \int_0^{\sqrt{1-y^2}} y^2 \, dx \, dy$
8. Evaluate : $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^x dz \, dy \, dx$
9. Find by double integration the area bounded between the curves $y^2 = 4x$ and $2x - 3y + 4 = 0$.
10. Evaluate : $\int_0^1 \int_0^{1-x} (x^2 + y^2) dx \, dy$
11. Evaluate : $\int_0^{\pi/2} \int_0^{2a\cos\theta} r \, dr \, d\theta$
12. Evaluate : $\iint xy(x + y) dx \, dy$ over the region enclosed by the parabolas $x^2 = y$, $y^2 = -x$
13. Change the order of integration by showing the region of integration :
$\int_{-a}^{a} \int_0^{y^2/a} f(x, y) dx \, dy$
14. Evaluate : $\iiint \frac{dx \; dy \; dz}{(x+y+z+1)^3}$ over the region bounded by the coordinate plane $x + y + z = 1$.
15. Find by double integration the area enclosed by the ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
16. Evaluate : $\iint e^{y^2} \, dx \, dy$ over the region bounded the triangle with vertices $(0,0), (2,1), (0,1)$.
17. Change the order of integration : $\int_0^8 \int_{\frac{y-8}{4}}^{y/4} f(x, y) dx \, dy$
18. Evaluate: $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}} \, dx \, dy$ over the first quadrant of the circle $x^2 + y^2 = 1$ by changing to polar coordinates.
19. Evaluate : $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dx \, dy \, dz$
20. Find the area common to the circles $x^2 + y^2 - 4y = 0$ and $x^2 + y^2 - 4x - 4y + 4 = 0$
21. Evaluate : $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx \, dy$
22. Evaluate : $\iint e^{\frac{y}{x}} \, dx \, dy$, over the area bounded by the curves $y = x^2$, $y = 0$ and $x = 1$.
23. Evaluate : $\int_0^{\pi/2} \int_x^{\pi/2} \int_0^{xy} \cos\frac{z}{x} \, dz \, dxy \, dx$
24. Find by double integration the area bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$
25. Evaluate: $\int_1^{\log 8} \int_0^{\log y} e^{x+y} \, dx \, dy$
26. Evaluate : $\iint y \, dx \, dy$ over the area bounded by $y = x^2$ and $x + y = 2$

27. Change the order of integration by showing the region of integration :
$\int_0^a \int_{x^2/a}^{2a-x} f(x, y) \, dx \, dy$
28. Evaluate : $\int_0^3 \int_{1/x}^{1} \int_0^{\sqrt{xy}} xyz \, dz \, dy \, dx$
29. Find the area between: $y^2 = \frac{x^3}{a-x}$ and its asymptotes.
30. Evaluate: $\iint xy \, dx \, dy$ over the region bounded by the parabola $x^2 = y$ and $y^2 = -x$
31. Find the double integration the area included between the cardioids :
$r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$
32. Change the order of integration by showing the region of integration :
$\int_0^a \int_{\sqrt{a^2-y^2}}^{y+a} f(x, y) dx \, dy$

33. Change to polar coordinates and evaluate : $\iint_R \frac{1}{\sqrt{xy}}$ dx dy where R is the region bounded by $x^2 + y^2 - x = 0, y = 0, y > 0.$

34. Change the order of integration and evaluate: $\int_0^1 \int_x^{1/x} \frac{y\,dxdy}{(1+xy)^2(1+y^2)}$

35. Evaluate : $\iint_A x^{m-1}y^{n-1}$ dx dy where A is bounded by $x + y = h, x = 0, y = 0.$

36. Evaluate : $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} x^2yz\,dx\,dy\,dz$

37. Evaluate : $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ over the region bounded by the coordinate plane $x + y + z = 7.$

38. Evaluate by changing to polar form : $\int_0^{\frac{a}{\sqrt{2}}} \int_y^{\sqrt{a^2-y^2}} \log(x^2 + y^2)\,dx\,dy, (a > 0)$

39. Change the order of integration : $\int_0^3 \int_y^{9/y} f(x, y)\,dxdy$

40. Evaluate : $\iint x\,y^2$ dxdy over the region bounded by $x = y^2, y = 1$ and $Y - \text{axis}$

41. Evaluate : $\int_0^3 \int_{y^2/9}^{\sqrt{10-y^2}} dy\,dx$

42. Change to polar coordinate and evaluate : $\iint \frac{(x^2+y^2)^2}{x^2y^2}$ dxdy over the region common to the circles $x^2 + y^2 = ax$ and $x^2 + y^2 = by\ (a, b > 0)$

43. Evaluate: $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dx\,dy\,dz}{(x+y+z+1)^3}$

44. Change to polar coordinate $\iint_R \sqrt{x^2 + y^2}$ dxdy, where R is the circle $x^2 + y^2 = 4.$

45. Evaluate : $\int_0^1 \int_1^2 xy\,dy\,dx$

46. Change the order of integration and evaluate : $\int_0^\pi \int_x^\pi \frac{\sin y}{y}\,dx\,dy$

47. Evaluate : $\int_0^1 \int_0^{1+x} (x - y)\,dxdy$

48. Change the order of integration by showing the region of integration :
$\int_0^1 \int_{x^2}^{\sqrt{2-x^2}} f(x, y)dxdy$

49. Evaluate : $\iint r^2 dr\,d\theta$ over the area included between $r = 2\sin\theta$, $r = 4\sin\theta.$

50. Evaluate : $\int_{-1}^1 \int_0^2 \int_{x-z}^{x+z} (x + y + z)dx\,dy\,dz$

51. Evaluate: $\int_0^a \int_y^b \frac{x^2}{(x^2+y^2)^{1/2}}\,dx\,dy$ by changing the order of integration [B.tech. May/June 2015]

52. Evaluate: $\int_0^3 \int_0^1 (x^2 + 3y^2)dx\,dy$ [B.tech. May/June 2015]

53. Change the order of integration: $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{y\,dx\,dy}{(1+y^2)\sqrt{(1-x^2-y^2)}}$ [B.tech. May/June 2015]

54. Change to polar coordinates: $\iint_R \sqrt{x^2 + y^2}\,dx\,dy$ where R: $x^2 + y^2 = 4$ [B.tech. May/June 2015]

55. Change the order of integration and evaluate: $\int_0^1 \int_y^{\sqrt{y}} xy\,dx\,dy$ [B.tech. May/June 2015]

56. Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dx\,dy\,dz$ [B.tech. May/June 2015]

57. Find the area between the curve: $y^2 = \frac{a^2x}{a-x}$ and its asymptote. [B.tech. May/June 2015]

58. Show that the surface area of the sphere generated by the revolution of the upper part of the circumference of the circle $x^2 + y^2 = a^2$ about x-axis is $4\pi a^2$
[B.tech. May/June 2015]

Unit-VI Fourier series

1. Find a_0 in the Fourier series of :
   $f(x) = 0 \ , -\pi \leq x \leq 0$
   $= \frac{\pi}{4} x, 0 \leq x \leq \pi$
2. Find a_0 in the Fourier series of $f(x) = e^x$ in $-\pi < x < \pi$.
3. Obtain the Fourier series of :
   $f(x) = -\pi \ , -\pi < x < 0$
   $= \pi, \quad 0 < x < \pi$
4. Obtain the Fourier series expansion for the function :
   $f(x) = 1 + x^2$ in $(-2,2)$
5. Obtain the Fourier series expansion for the function :
   $f(x) = 1 - \frac{2x}{\pi} \ , \quad -\pi < x < 0$
   $= 1 + \frac{2x}{\pi} \ , \quad 0 < x < \pi$
6. Find half range cosine series for : $f(x) = x \sin x$ in $0 < x < \pi$
7. Express the function $f(x) = \frac{1}{2} (\pi - x) , 0 < x < 2\pi$ in Fourier series
8. Find sine expansion of $lx - x^2$ in $(0, l)$
9. If $f(t) = 1 - t^2$ find Fourier series of $f(t)$, $-1 \leq t \leq 1$
10. Express $f(x) = 2 - x$ in Fourier series for $(0,2)$, $f(x) = f(x + 2)$
11. In the cosine series of :
    $f(x) = 1 \ , \quad 0 < x < 1$
    $= \pi, \quad 1 < x < 2$ Find the value of a_0
12. Find the value of a_0 in the Fourier series for $f(x) = |x|$ in $(-\pi, \pi)$
13. Find the Fourier series for the function $f(x) = (\frac{\pi - x}{2})^2$ in $0 < x < 2\pi$
14. Find Fourier series for :
    $f(x) = -x \ , -\pi < x < 0$
    $= 0 \ , \quad 0 < x < \pi$
15. Obtain the Fourier series expansion for the function :
    $f(x) = \cos x \ , \quad -\pi < x < 0$
    $= -\cos x \ , \quad 0 < x < \pi$
16. Find half range sine series for :
    $f(x) = x \ , \quad 0 \leq x \leq l/2$
    $= l - x, \quad l/2 < x < l$
17. Define Fourier series of $f(x)$ in $(0,2\pi)$
18. If Fourier series of $f(x) = x$ in $(-\pi, \pi)$ is : $f(x) = 2 \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n} \ sn(nx)$
    then prove that: $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

19. Define Dirichlets conditions.
20. Find the Fourier series of the function x^2 in $(0, a)$
21. Find the Fourier series of the function :

$$f(x) = x , \quad 0 \leq x \leq \pi$$
$$= 2\pi - x , \quad \pi \leq x \leq 2\pi$$

22. Find the Fourier series expansion of $\cosh x$ in $-\pi$ to π
23. Express the function $f(x) = \frac{1}{2}(\pi - x) , 0 < x < 2\pi$ in Fourier series
24. Express $f(x) = 2 - x$ in Fourier series for $(0,2)$, $f(x) = f(x + 2)$
25. If $f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$, $(0,2\pi)$ prove that $f(x) = \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ and hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$
26. Find Fourier series of $f(t) = 0, \; 0 < x < \pi$

$$= 1 , \; \pi < x < 2\pi$$

27. Find Fourier series of $f(t) = a \sin t, \; 0 \leq t \leq \pi$

$$= 0 , \; \pi \leq x \leq 2\pi$$

28. Find Fourier series of $\cos x$ over $(0,2\pi)$
29. Find Fourier series of $x \cos x$ over $(0,2\pi)$
30. Prove that: $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2} , -\pi < x < \pi$. Hence show that $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$
31. Find Fourier series of $|\cos x|$ over $(-\pi, \pi)$
32. Find Fourier series of $\frac{100x}{l}$ over $(-l, l)$
33. Obtain a Fourier expression for $f(x) = x^3 , -\pi < x < \pi$
34. Find Fourier series of $x \cos x$ over $(-\pi, \pi)$
35. Find Fourier series of $f(x) = k(x - l), (-l, 0)$

$$= k(x + l), (0, l)$$

36. Find Fourier series of $f(x) = -x , -4 < x < 0$

$$= x, \quad 0 < x < 4$$

37. Find Fourier series of $f(x) = x + \pi , 0 < x < \pi$

$$= x - \pi , -\pi < x < 0$$

38. Find Fourier series of $x + x^2$ over $(-\pi, \pi)$.Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$
39. Find Fourier series to represent $x - x^2$ from $x = -\pi$ to π and show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

40. Find Half range cosine series for $f(x) = x$ in $0 < x < 2$

41. Find Fourier sine series of

$$f(x) = x, 0 < x < 4$$

$$= 8 - x, 4 < x < 8$$

42. Find Fourier cosine series of

$$f(x) = 1, (0,1)$$

$$= x, (1,2)$$

43. Find half range sine series for $f(x) = 1$ in $0 < x < \pi$ Hence show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$

44. Find Half range cosine series for $f(x) = \pi x, \ 0 < x < 1$

$$= \pi(2 - x), 1 < x < 2$$

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