Function of complex variable & its applications

Question Bank

Engineering Mathematics-IV (B.tech)

1. If f(z) is analytic in a closed curve c,except at a finite number of singular points within c, then: $\int_c ¿\, f(z)\,dz = ¿$ ……. [B.tech.Nov/Dec 2012]
2. The value of $\int_0^{\pi i} ¿\, \cos z\, dz = ¿$ …….. [B.tech.Nov/Dec 2012]
3. If f(z) has a pole of order three at $z=a$ then: $Res f(a) = \ldots$ [B.tech.Nov/Dec 2012]
4. If C is the circle |z| $¿2$ then $\int_c ¿\, \frac{1}{z-1}dz = ¿$ …… [B.tech.Nov/Dec 2012]
5. Prove that $u = x^2 - y^2 \wedge v = \frac{y}{x^2+y^2}$ are harmonic functions of (x,y) but not harmonic conjugates. [B.tech.Nov/Dec 2012]
6. Evaluate: $\int_c \frac{2z^2+z}{z^2-1}dz$ where c is | $z-1 \vee ¿ 1$ by Cauchy integral formula. [B.tech.Nov/Dec 2012]
7. Evaluate $\int_c ¿\, z \vee^2 dz$ around the square with vertices at (0,0),(1,0),(1,1) and (0,1). [B.tech.Nov/Dec 2012]
8. Find the image of the circle |z| $¿2$ under the transformation w $¿\, z+3+2$i. [B.tech.Nov/Dec 2012]
9. If f(z) is analytic within and on a closed curve and if ‘a’ is any point within c, then

$\frac{1}{2\pi i}\int_c \frac{f(z)}{z-a} = ¿$ …….. [B.tech. May/June 2012]

10. The value of $\int_0^{2i} ¿$ $sinhz\, dz = ¿$ ……. [B.tech. May/June 2012]

11. The poles of the function $f(z)=\frac{z^3-1}{z^3+1}$ are ….. [B.tech. May/June 2012]

12. If C is the circle |z| $¿2$ then $\int_c \frac{e^z}{(z-3)^2} dz = ¿$ …… [B.tech. May/June 2012]

13. Let $f(z)=u+iv$ be an analytic function, if $u=3x-2xy$ then find v and express f(z) in terms of z. [B.tech. May/June 2012]

14. Evaluate: $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle| $z \vee ¿ 2$ by Cauchy integral formula.

[B.tech. May/June 2012]

15. Find the image of the lines :

i) $x=y+1$

ii) The line joining A (1+i) to B (2+3i) in z-plane under the transformation W= $\frac{i}{z}$

[B.tech. May/June 2012]

16. Evaluate: $\int_0^{\infty} \frac{dx}{(1+x^2)^2}$ using residue theorem.

17. If $u-v=(x-y)(x^2+4xy+y^2)$ and $f(z)=u+iv$ is an analytic function at $z=x+iy$, find f (z) in terms of z.

18. Evaluate: $\int_c \frac{z}{(z-1)(z-3)} dz$ where c is the circle| $z \vee ¿ 3$ [B.tech. May/June 2011]

19. If f(z) is analytic at all points within C in a multiple connected region R then,

$\int_c f(z)\,dz =$ .... [B.tech. May/June 2012]

20. If C is the circle $|z| = \frac{1}{2}$ then $\int_c \frac{e^{-z}}{z+1}\,dz$ =....... [B.tech. May/June 2011]

21. If C is the circle $|z| = 1$ then $\int_c \frac{e^{-z}}{z^2}\,dz$ =....... [B.tech. May/June 2011]

22. Evaluate: $\int_c \frac{3z^2+z}{z^2-1}\,dz$ where c is $|z-1| = 1$ [B.tech. May/June 2011]

23. Cauchy-Riemann equation in Cartesian coordinates are......

[B.tech. Oct/Nov 2011]

24. If f(z) is analytic function and f'(z) is continuous at each point within and on a closed curve C,then $\int_c f(z)\,dz =$ .... [B.tech.Oct/Nov 2011]

25. If $f'(z) = \cos x \cosh y - i \sin x \sinh y$ then $f(z) =$ ....

[B.tech.Oct/Nov 2011]

26. If C is the circle $|z-1| = 1$ then $\int_c \frac{\cos z}{z}\,dz$ =....... [B.tech.Oct/Nov 2011]

27. Find p such that the function: $f(z) = r^2 \cos 2\theta + i r^2 \sin p\theta$ is analytic.

[B.tech.Oct/Nov 2011]

28. Find the analytic function: $f(z) = u + iv$ given $r = a(1+\cos\theta)$

29. Evaluate: $\int_c \frac{e^z}{\cos \pi z}\,dz$ where C is the circle $|z| = 1$ [B.tech. May/June 2010]

30. If f (z) is analytic within and on a closed curve C then $\int_c \frac{f(z)}{z-a}\,dz$ =.......

[B.tech. May/June 2010]

31. If f (z) is a ingle valued and analytic within and on a closed curve C then $\int_c \frac{f(z)}{(z-a)^{n+1}} dz$ =……. [B.tech. May/June 2010]

32. Every analytic function $f(z)=u(x,y)+iv(x,y)$ defines two families of curves $u(x,y)=C_1 \wedge v(x,y)=C_2$ .These family of curves are……….to each other. [B.tech. May/June 2010]

33. Prove that the function $coshz$ is analytic and find its derivative. [B.tech. May/June 2010]

34. If C is the circle | $z \vee ¿ 2$ then evaluate $\int_c \frac{e^{-z}}{z+1} dz$ [B.tech. May/June 2010]

35. The necessary and sufficient condition for the function $w=f(z)=u(x,y)+iv(x,y)$ to be analytic in a region R, are…….. [B.tech. May/June 2010]

36. An electrostatic field in the xy plane is given by the potential function : ɸ $¿ 3x^2 y - y^3$. Find the stream function. [B.tech. May/June 2011]

37. Evaluate: $\int_c \frac{2z-1}{z(z+1)(z-3)} dz$ where c is the circle| $z \vee ¿ 2$ [B.tech. May/June 2010]

38. If C is the circle | $z-1 \vee ¿ 3$ then $\int_c \frac{cosz}{z-\pi} dz$ =……. [B.tech. May/June 2010]

39. Determine p such that the function $f(z)=\frac{1}{2}\log(x^2+y^2)+itan^{-1}\frac{px}{y}$ is analytic function. [B.tech.(Old) May/June 2010]

40. Determine analytic function $w=u+iv$ if $v=\log(x^2+y^2)+x-2y$ [B.tech.(Old) May/June 2010]

41. Cauchy-Riemann equation in polar coordinates are……. [B.tech. Oct/Nov 2009]

42. C-R equation in Cartesian form = …….. [B.tech. Oct/Nov 2009]

43. Residue of $e^{\frac{1}{z^2}} cosz = ¿$ ……. [B.tech. Oct/Nov 2009]

44. Residue of $\frac{tanz}{z}$ at $z=0$ is…… [B.tech. Oct/Nov 2009]

45. If f(z) has a pole of order m at $z=a$ then the residue of f(z) at $z=a$ is…… [B.tech. Oct/Nov 2009]

46. Prove that an analytic function with constant amplitude is constant. [B.tech. Oct/Nov 2009]

47. Evaluate: $\int_c \frac{e^{3z}}{z-\pi i} dz$ where C is the ellipse $|z-2|+|z+2|=6$

[B.tech. Oct/Nov 2009]

48. If $f(z)=u+iv$ analytic function, find f (z) if $u+v=r^2(cos2\theta+sin2\theta)$ when f (0) =0. [B.tech. Oct/Nov 2009]

49. Evaluate using residue theorem $\int_C \frac{z-3}{z^2+2z+5} dz$ where C:| $z+1-i\vee¿3.$

[B.tech. Oct/Nov 2009]

50. Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle | $z-2i\vee¿\frac{3}{2}$

[B.tech.May/June 2009]

51. If $u+v=e^x(cosy+siny)+\frac{x-y}{x^2+y^2}$, find the analytic function $f(z)=u+iv$

Where f(1)=1. [B.tech.May/June 2009]

52. Evaluate: $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2}$ [B.tech. May/June 2009]

53. Find Laurent's expansion for $f(z)=\frac{1}{(1-z^2)(z+2)}$

For i) $¿z\vee¿1$ ii) 1 $¿\vee z\vee¿$ <2 [B.tech. May/June 2009]

54. Find an analytic function $f(z)$ such that

$\Re[f'(z)]=3x^2-4y-3y^2$ and $f(1+i)=0$ [S.E.Oct/Nov 2011]

55. Evaluate: $\int_1^i \frac{\log^3 z}{z} dz$ along the arc of the circle $|z|=1$ [S.E.Oct/Nov 2011]

56. Prove that $u = r^2 \cos 2\Phi$ is harmonic function and find its harmonic conjugate and also corresponding analytic function. [S.E.Oct/Nov 2011]

57. Evaluate $\int_C \frac{\sin z}{(z-1)^2(z^2-9)} dz$ where C is the circle :

(i) $|z-3i|=1$ (ii) $|z-3|=\frac{1}{2}$ [S.E.Oct/Nov 2011]

58. If $f(z)=\frac{z+4}{(z+3)(z-1)^2}$ find Laurent's series expansion in

(i) $0<|z-1|<4$ (ii) $|z-1|>4$ [S.E.Oct/Nov 2011]

59. Evaluate by Cauchy's residue theorem $\int_C \frac{1}{z \sin z} dz$ where C is the circle

$|z|=\frac{1}{2}$

60. Find the image of the square with vertices (0,0) , (2,0) (2,2) (0,2) under the transformation $W=(1+i)z+(2+i)$ [S.E.Oct/Nov 2011]

61. Evaluate: $\int_0^{2\pi} \frac{2}{2+\cos\theta} d\theta$ [S.E.Oct/Nov 2011]

62. Find the bilinear transformation which maps the points $1,-1,\infty$ in z-plane on to the points $1, i, -1$ in w-Plane. [S.E.Oct/Nov 2011]

63. Evaluate: $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2} (a>0)$ [S.E.Oct/Nov 2011]

64. Determine whether the function $f(z)=z e^{-z}$ is analytic or not. [S.E.Nov/Dec 2012]

65. Evaluate: $\int_0^{\pi-\pi i} e^z dz$ along the curve C: $x=t, y=-t$ [S.E.Nov/Dec 2012]

66. Prove that: $v=r^2 \sin 2\phi - \frac{1}{r}\sin\phi$ is harmonic and find its harmonic conjugates and also corresponding analytic function. [S.E.Nov/Dec 2012]

67. Evaluate: $\int_c \frac{z^2}{z^4-1}dz$ where C is the circle

(i) $|z|=\frac{1}{2}$ (ii) $|z-1|=\frac{1}{2}$ [S.E.Nov/Dec 2012]

68. Expand $\cos z$ in to a Taylor's series about the point $z=\frac{\pi}{2}$ [S.E.Nov/Dec 2012]

69. Evaluate by Cauchy's residue theorem: $\int_c \frac{z}{\sin^2 z}dz$ where C: $|z|=\frac{1}{5}$

[S.E.Nov/Dec 2012]

70. Find the bilinear transformation which maps the points $2, i, -2$ in z-plane on to the points $1, i, -1$ in w-Plane. [S.E.Nov/Dec 2012]

71. Evaluate: $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$ [S.E.Nov/Dec 2012]

72. Find the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$

[S.E.May/June 2012]

Evaluate: $\int_C \Re(z)\,dz$ where C is the semi unit circle. [S.E.May/June 2012]

73. Evaluate by using Cauchy's integral formula: $\int_C \frac{z+1}{z^2+2z+4}dz$ where C is $|z+1+i|=2$ [S.E.May/June 2012]

74. Evaluate: $\oint_C \frac{\coth z}{z-i}dz$ where C is $|z|=2$ by Cauchy's residue theorem.

[S.E.May/June 2012]

75. Expand $f(z)=\frac{7z-2}{z(z+1)(z-2)}$ for $1<|z+1|<3$ [S.E.May/June 2012]

76. Find the bilinear transformation which maps the points $z=-1,2,i-1$ in to the points $W=\infty,\frac{2i}{3},i-1$. [S.E.May/June 2012]

77. Find the analytic function whose real part is $y+e^x cosy$ [S.E.Nov/Dec 2008]

78. Evaluate: $\int_C (z-z^2)dz$ where C is upper half of the circle $|z|=1$ [S.E.Nov/Dec 2008]

79. Apply Cauchy's residue theorem to evaluate: $\oint_C \frac{dz}{(z^2+1)(z^2+4)}$ where C is the circle $|z|=1.5$ [S.E.Nov/Dec 2008]

80. Determine the bilinear transformation that maps the points: $1-2i,2+i,2+3i$ respectively into $2+2i,1+3i,4.$ [S.E.Nov/Dec 2008]

81. Use Cauchy's integral formula to evaluate: $\int_C \frac{z^2-2z+1}{(z-i)^2}dz$ where C is $|z|=2$ [S.E.Nov/Dec 2008]

82. Prove that $u=\log(x^2+y^2)$ is harmonic function and find its harmonic conjugate and also corresponding analytic function. [S.E.Nov/Dec 2008]

83. If the real part of an analytic function $f(z)$ is x^2-y^2-y then find imaginary part. [S.E.Nov/Dec 2014]

84. If $f(z)=\frac{a}{2}rcos\theta+i(rsin\theta+2)$ is harmonic, then find the value of $a.$ [S.E.Nov/Dec 2014]

85. Find the imaginary part of analytic function whose real part is $r^{-4}cos4\theta$

86. Find whether the function $u=\log ¿ z \vee ¿^2$ is harmonic. If so find analytic function whose real part is u . [S.E.Nov/Dec 2014]

87. Show that the image of the line $x=0$ under the transformation $w=e^z$ is a circle.

88. Evaluate $\int_0^{1+i} (x+iy)\,dz$ along $y=x$ [S.E.Nov/Dec 2014]

89. Evaluate $\int_C (z-i)\,dz$ where C is 0 to i [S.E.Nov/Dec 2014]

90. Find the poles of $f(z)$ and residues at the poles which lie on imaginary axis if $f(z)=\frac{z^2-2z}{(z+1)(z^2+4)}$ [S.E.Nov/Dec 2014]

91. Find the Bilinear transformation which maps the points $z=1,-i,1$ onto the points $w=2,i,-2$ [S.E.Nov/Dec 2014]

92. Evaluate $\int_{(0,0)}^{(1,1)} (3x^2+4xy+ix^2)\,dx$ along $y=x^2$ [S.E.Nov/Dec 2014]

93. Expand $f(z)=\frac{1}{(z-1)(z-2)}$ for $0<|z-1|<1$ [S.E.Nov/Dec 2014]

94. Evaluate $\oint_C \frac{dz}{\sinh^2 z}$ where C is $|z|=2$ by using Cauchy's integral formula.

95. Under the transformation $W=z+\frac{a^2-b^2}{4z}$ maps the circle of radius $\frac{1}{2}(a+b)$ with centre at origin in the z-plane into an ellipse on W-plane.

96. Evaluate $\oint_C \frac{coshz}{(z+1)^3(z-1)}dz$ where C is $|z|=2$ by Cauchy's Residue theorem.

97. Evaluate: $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ using residue theorem. [S.E.Nov/Dec 2014]

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