

# Differential Equation

## Question Bank

### Engineering Mathematics-II (B.tech)

1. Solve:  $\frac{(2xy+1)}{y} dx + \frac{y-x}{y^2} dy = 0$
2. Solve:  $(1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1}x}$
3. Find the orthogonal trajectories of the family of curve:  $r = a(1 - \cos \theta)$
4. Solve:  $\frac{dy}{dx} - xy = y^2 e^{-x^2/2} \log x$
5. A resistance of 100 ohms, an inductance of 0.5 Henry is connected in series with a battery of 20 volts. Find the current in a circuit as a function of t.
6. A particle falls under gravity in a resisting medium of which the resistance varies as the velocity. If the particle starts from rest, find the velocity at any time t.
7. Solve:  $\left(1 + 2e^{\frac{x}{y}}\right) dx + 2e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$
8. Solve:  $\sin 2x \frac{dy}{dx} = y + \tan x$
9. Find the orthogonal trajectories of:  $\left(r + \frac{k^2}{r}\right) \cos \theta = \alpha$
10. Solve:  $\sin y \frac{dy}{dx} = (1 - x \cos y) \cos y$
11. The equation of the electromotive force in terms of current  $i$  for an electrical circuit having resistance  $R$ , and a condenser of capacity  $C$  in series is:  $E = Ri + \int \frac{i}{C} dt$   
 $E = \dot{i}$   
Find the current  $i$ , when  $E = E_m \sin \omega t$ .
12. A particle of mass  $m$  is projected vertically upwards under gravity, the resistance of the air being  $mk$  times the velocity. Show that the greatest height attained by the particle is:

$\frac{v^2}{g}[\lambda - \log(1 + \lambda)]$  where v is the greatest velocity which above mass will attain when

it falls freely and  $\lambda v$  is the initial velocity.

13. Solve:  $\left[ \cos x \log_e(2y-8) + \frac{1}{x} \right] dx + \frac{\sin x}{y-4} dy = 0$  with  $y(1) = \frac{9}{2}$

14. Solve:  $x(x-1) \frac{dy}{dx} - (x-2)y = x^2(2x-1)$

15. Find the orthogonal trajectories of :  $r^n = a^n \cos n\theta$

16. An e.m.f. is connected in series with resistance R and inductance L, where  $L=640, R=250, E=500$ .

i) Form the differential equation for the circuit.

ii) Show that current will approach 2 amps as t increases.

iii) Find in how many seconds i will approach 90% of its maximum value.

17. A body of mass m falling from rest is subjected to the force of gravity and air resistance of k times of (velocity)<sup>2</sup>. If it falls through a distance x and possesses a velocity v at

that instant, prove that :  $\frac{2kx}{m} = \log \left( \frac{a^2}{a^2 - v^2} \right)$  where  $mg = k a^2$

18. Solve:  $(1 + \sin y) \frac{dx}{dy} = 2y \cos y - x (\sec y + \tan y)$

19. Solve:  $(2x y^4 + \sin y) dx + (4 x^2 y^3 + x \cos y) dy = 0$

20. Find the orthogonal trajectories of the family of curve:  $r = \frac{2a}{1 + \cos \theta}$

21. Solve:  $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$

22. Solve:  $3x(1 - x^2)y^2 \frac{dy}{dx} + (2x^2 - 1)y^3 = a x^3$

23. Solve:  $\frac{dy}{dx} + 2y \tan x = y^2 \tan^2 x$

24. A particle of mass m under gravity in a medium whose resistance is k times velocity where k is constant. If the particle is projected vertically upwards with velocity V, show that the time to reach the highest point is :

$$\frac{m}{k} \log \left[ 1 + \frac{KV}{mg} \right]$$

25. Under what condition the equation:  $(\cosh y + \cos x) dx + b x \sinh y dy = 0$  is exact?

26. Find the integrating factor of:  $\sec^2 y \frac{dy}{dx} + x \tan y = x^3$

27. Solve:  $(x+1) \frac{dy}{dx} - y = e^{3x} (x+1)^2$

28. Solve:  $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

29. Solve:  $r \sin \theta - \frac{dr}{d\theta} \cos \theta = r^2$

30. A particle falls under gravity in a resisting medium whose resistance varies as the velocity. If the particle starts from rest; find the distance travelled by the particle in time  $t$ .

31. Given  $L \frac{di}{dt} + Ri = E$ :

i) Find current  $i$

ii) Show that current will approach 2 amps as  $t$  increases. (when  $L=540, R=150, E=300$ )

iii) Find in how many seconds  $i$  will approach 90% of its maximum value

32. Solve:  $e^{-y} \sec^2 y dy = dx + x dy$

33. Solve:  $\frac{dy}{dx} = -\left(\frac{x+y \cos x}{1+\sin x}\right), y\left(\frac{\pi}{2}\right) = 1$

34. Find the orthogonal trajectories of the family of curve:  $r^m = a^m \cos m\theta$

35. Solve:  $y^2 \frac{dx}{dy} + xy = 2y^2 + 1$

36. Solve:  $\frac{dy}{dx} = \frac{y(y-e^x)}{e^x - 2xy}$

37. Find the orthogonal trajectories of the family of curve:  $a y^2 = x^3$

38. A particle is projected vertically upwards with velocity  $V_1$  and resistance of the air

produces retardation  $K V^2$ , where  $V$  is the velocity. Find the greatest height attained by the particle.

39. Solve:  $y e^x dx = (y^3 + 2x e^y) dy$

40. Solve:  $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$

41. Solve:  $(1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1}x}$

42. In an electric circuit containing resistance R, an inductance L, the voltage E and current i

Are connected by the equation:  $E = Ri + L \frac{di}{dt}$ , If  $L=320, R=150, E=450$  and  $i=0$  when

$t=0$ . show that the current i will approach 3 amp as t increases.

43. A moving body is opposed by a force per unit mass of value cx and resistance per unit mass of value  $bv^2$ , where x and v are the displacement and velocity of the particle at that instant. Find the velocity of the particle in terms of x, if it starts from rest.

44. Define exact differential equation.

45. Find the integrating factor of:  $(1+y^2)dx = (\tan^{-1}y - x)dy$

46. Solve:  $3y^2 \frac{dy}{dx} + 2y^3x = 4xe^{-x^2}$

47. Solve:  $(1+2xy \cos x^2 - 2xy)dx + (\sin x^2 - x^2)dy = 0$ .

48. Solve:  $y \log y dx + (x - \log y)dy = 0$

49. Define linear differential equation and Bernoulli's differential equation.

50. Solve:  $y \frac{dx}{dy} - x = 2y^3$  [F.E. Nov/Dec 2013]

51. Solve:  $\frac{dy}{dx} + y \tan x = y^3 \sec x$  [F.E. Nov/Dec 2013]

52. Solve:  $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$  [F.E. Nov/Dec 2013]

53. Find the orthogonal trajectories of the parabola:  $y^2 = 4ax$  [F.E. Nov/Dec 2013]

54. Find the orthogonal trajectories of the curve:  $xy = c$  [F.E. Nov/Dec 2013]

55. Find the solution of exact differential equation:  
 $(x+2y-2)dx + (2x-y+3)dy = 0$  [F.E. April/May 2012]

56. Solve:  $(\sec x \tan x \tan y - e^x)dx + (\sec x \sec^2 y)dy = 0$  [F.E. April/May 2012]

57. Solve:  $\tan y \frac{dy}{dx} - \cos y \cos^2 x = -\tan x$  [F.E. April/May 2012]

58. Solve the equation:  $L \frac{di}{dt} + Ri = 20 \cos(3t)$  where  $R=10$  ohms,  $L=0.5$  henry.

Given that  $i=0$  when  $t=0$ . [F.E. April/May 2012]

59. Solve:  $\frac{dx}{dy} = \frac{2xy}{x^2 - y^2}$  [F.E.Nov/Dec 2008]

60. Solve:  $\sin x \frac{dx}{dt} - \cos x + t \cos^2 x = 0$  [F.E.Nov/Dec 2008]

61. Solve:  $\cos^2 x \frac{dy}{dx} - \tan x = -y$  [F.E.Nov/Dec 2008]

62. A condenser of capacity  $c$  is charged through a resistance  $R$  by steady voltage  $V$ , show

that the charge  $q$  on the plate is given by:  $R \frac{dq}{dt} + \frac{q}{c} = V$  hence show that if  $q=0$  at

$t=0$ ,  $q = cv \left[ 1 - e^{-\frac{t}{RC}} \right]$  [F.E.Nov/Dec 2008]

63. Solve:  $(x+a) \frac{dy}{dx} - 3y = (x+a)^5$  [F.E.May/June 2008]

64. Solve:  $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$  [F.E.May/June 2008]

65. Solve:  $(y^2 e^{xy^2} + 4x^3) dx + (2xy e^{xy^2} - 3y^2) dy = 0$  [F.E.May/June 2008]

66. Find the orthogonal trajectories of the family of curve:  $x^2 + cy^2 = 1$

[F.E.May/June 2008]

67. A circuit containing resistance of 20 ohms and all inductance 10 henries is connected to 100 volts supply. Determine current after 2 seconds. [F.E.May/June 2008]

68. Show that  $\frac{g}{n^2} \log (\cosh nt)$  is the distance passed over by a body falling vertically

from rest, assuming that the resistance of air is  $\frac{n^2}{g}$  times the square of the velocity.

[F.E.May/June 2008]

69. Solve:  $(x + \tan y) dy = \sin 2y dx$  [F.E.Nov/Dec 2007]

70. Solve:  $x dx + y dy = \frac{a(x dy - y dx)}{x^2 + y^2}$  [F.E.Nov/Dec 2007]

71. Solve:  $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$  [F.E.Nov/Dec 2007]

72. Find the orthogonal trajectories of the family of curve:  $r = a(1 + \cos \theta)$

[F.E.Nov/Dec 2007]

73. The equation of L-R series circuit is given by  $L \frac{di}{dt} + Ri = 4 \sin 3t$  if  $i=0$  at  $t=0$

then express  $i$  as function of  $t$ . [F.E.Nov/Dec 2007]

74. Find the integrating factor of  $(1+y^2)dx = [\tan^{-1}y - x]dy$  [F.E.Nov/Dec 2013]

75. Solve:  $[\cos x \tan y + \cos(x+y)]dx + [\sin x \sec^2 x + \cos(x+y)]dy = 0$

[F.E.Nov/Dec 2007]

76. Solve:  $y \frac{dx}{dy} - x = 2y^2$

[F.E.Nov/Dec 2013]

77. Find the orthogonal trajectories of the family of curve:  $r^2 = c \sin 2\theta$

[F.E.Nov/Dec 2013]

78. Find the integrating factor of:  $R \frac{dQ}{dt} + \frac{Q}{C} = V$

[B.Tech Nov/Dec

2013]

79. A constant emf  $E$  volts is applied to an electrical circuit containing resistance  $R$  and inductance  $L$  in series. If the initial current is zero show that the time for current to build

up to half of its maximum is:  $\frac{L \log 2}{R} \text{ sec}$ . [B.Tech Nov/Dec

2013]

80. A particle falls in a vertical line under gravity and air resistance to its motion is proportional to its velocity and distance as function of  $t$ . Show that the velocity  $V$  will

never exceed  $\frac{g}{k}$ .

[B.Tech Nov/Dec

2013]

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