Application Partial differential equation

Question Bank

Engineering Mathematics-IV (B.tech)

1. The solution of $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial y}$ is $u=¿$ ……

2. The solution of $\frac{\partial^2 z}{\partial x \partial y}=cotx\,cosy$ is z ……..

3. The solution of : $\frac{\partial u}{\partial x}=3\frac{\partial u}{\partial t}+u$ is $u=¿$ …….

4. If $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends ,

   find the solution with conditions :
   $y(0,t)=y(L,t)=0, y(x,0)=y_0 \sin\frac{\pi x}{L}$ and $\frac{\partial y}{\partial t}=0$ at $t=0.$

5. Solve : $\frac{\partial u}{\partial t}=4 \quad \frac{\partial^2 u}{\partial x^2}$ subject to the conditions,

   $u(0,t)=u(\pi,t)=0, u(x,0)=2\sin 3x-4\sin 5x.$

6. Solve $\frac{\partial u}{\partial x}-\frac{\partial u}{\partial t}=5u$, when $u(x,0)=5e^{-3x}+7e^{2x}$

7. Consider the conduction of heat along a bar which is covered by a material impervious to heat under the condition :

   i) V $\neq \quad \infty$ as t $\to \infty$

   ii) $\frac{\partial v}{\partial x}=0$ when $x=0 \wedge x=l$

   iii) $V=lx-x^2 w\,h\,en\,t=0\,between\,x=0 \wedge x=l.$ then solve $\frac{\partial v}{\partial t}=k\frac{\partial^2 v}{\partial x^2}$

8. Solve the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ subject to the conditions :

   (i) $u=0\,w\,h\,en\,y\to\infty$

   (ii) $u=0\,w\,h\,en\,x=0$

(iii) $u=0\ when\ x=1$

(iv) $u=x(1-x)\ when\ y=0\ for\ 0<x<1.$

9. Solution of $\frac{\partial^2 z}{\partial x \partial y}=secx\,cosy$ is z ……..

10. Solution of $\frac{\partial^2 z}{\partial x \partial t}=e^{2t-3x}$ is z ……..

11. Solution of $\frac{\partial^2 z}{\partial x \partial y}=x^2 cosy$ is z ……..

12. A tightly stretched string with fixed end points $x=0$ and $x=L$ is initially in a position given by $y(x,0)=y_0\sin^3\left(\frac{\pi x}{L}\right)$ if it is released from rest position, find the displacement y at any distance x from one end and at any time t.

13. Solution of $\frac{\partial u}{\partial x}-\frac{\partial u}{\partial t}=0\ is\ u=\ldots$

14. Solution of $\frac{\partial^2 u}{\partial x \partial y}=cosecx\,secy$ is u ……..

15. Solution of $\frac{\partial^2 z}{\partial x \partial y}=cotx\,coty$ is z ……..

16. If $\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends , find the solution with conditions :

$y(0,t)=y(L,t)=0, \frac{\partial y}{\partial t}=0$ at $t=0.\ y(x,0)=y_0\sin\frac{\pi x}{2}$.

17. Solve : $\frac{\partial u}{\partial t}=4$ $\frac{\partial^2 u}{\partial x^2}$ subject to the conditions,

$u(0,t)=u(\pi,t)=0, u(x,0)=2\sin 4x-3\sin 2x$

18. Solution of $\frac{\partial^2 u}{\partial x \partial y}=e^{-x}cosy$ is u ……..

19. Solution of $\frac{\partial^2 u}{\partial x \partial t}=x^2 e^{2t}$ is u ……..

20. Solution of $\frac{\partial^2 u}{\partial x \partial y} = x^2 y^2$ is u ……..

21. Solve $\frac{\partial^2 z}{\partial x^2} = 4 \frac{\partial^2 y}{\partial y^2}$ satisfying the conditions $z = \frac{\partial z}{\partial y} = sinx$ when $y = 0.$

22. A rod of length L with insulated sides is initially at uniform temperature u_0 , its ends are suddenly cooled to 0^0 C and are kept at that temperature. Find the temperature function $u(x, t).$

23. Solution of $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} cosx$ is u ……..

24. Solution of $\frac{\partial^2 z}{\partial x \partial y} = sinx\, siny$ is z ……..

25. Solution of $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0 is\, u = \ldots$

26. Solve $\frac{\partial^2 z}{\partial x^2} = 4 \frac{\partial^2 y}{\partial y^2}$ satisfying the conditions $z = \frac{\partial z}{\partial y} = cosx$ when $y = 0.$

27. The one dimensional wave equation is represented by partial differential equation ……

28. The Solution of $3 \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0 is\, u = \ldots$

29. The Solution of $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u\, is\, u = \ldots$

30. Solve : : $\frac{\partial u}{\partial t} = a^2 \quad \frac{\partial^2 u}{\partial x^2}$ where $u(x, t)$ satisfies the following conditions :

(i) $u\, 0, t ¿ = 0$

(ii) $\frac{\partial u}{\partial x}_{(l,t)} = 0\, for\, all\, t$

(iii) $u(x, 0) = 0, 0 < x < l$

(iv) $u(x, \infty)$ is finite.

31. If $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibration of the string of length L , fixed at both ends , find the solution with conditions :

(i) $y(0,t)=0$

(ii) $y(l,t)=0$

(iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0}=0$

(iv) $y(x,0)=k(lx-x^2), 0<x<l.$

32. Solve: $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ by method of separation of variables.

33. Solve the boundary value problem: $\frac{\partial^2 y}{\partial t^2}=4\frac{\partial^2 y}{\partial x^2}$, $y(0,t)=y(5,t)=0, y(x,0)=0$

and $\frac{\partial y}{\partial t}=5\, sin\pi x$ when $t=0$. [S.E.Oct/Nov 2011]

34. Solve the boundary value problem: $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$ subject to the conditions:

$\frac{\partial u}{\partial x}(0,t)=0, \frac{\partial u}{\partial t}(5,t)=0, u(x,0)=5x-x^2, 0<x<5$ and u is not infinite for

$x\to\infty$.

[S.E.Oct/Nov 2011]

35. Solve: $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}=u, u(x,0)=3e^{-5x}+2e^{-3x}$ [S.E.Oct/Nov 2011]

36. Solve the boundary value problem: $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$

Subject to the conditions:

(i) $y(0,t)=0$

(ii) $y(l,t)=0$

(iii) $\frac{\partial y}{\partial t}=0$, when $t=0$

(iv) $y(x,0)=\frac{x}{100}(l^2-x^2)$ for $0\le x\le l$ [S.E.Oct/Nov 2012]

37. Solve: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ subject to the conditions $u(0,y)=u(l,y)=u(x,0)=0$ and

$u(x,a)=\sin\left(\frac{n\pi x}{l}\right)$ [S.E.Oct/Nov 2011]

38. Solve the boundary value problem: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<1, t>0$

Subject to the conditions:

(i) $u(0,t)=0$

(ii) $u(1,t)=0$

(iii) $u(x,0)=3 \sin m\pi x$ [S.E.Oct/Nov 2012]

39. Solve: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ within the rectangle $0\le x\le a, 0\le y\le b$ given that

$u(0,y)=0, u(a,y)=0, u(x,b)=0$ and $u(x,0)=x(a-x)$ [S.E.Oct/Nov 2012]

40. Solve: $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+3u$ subject to the conditions $u=0, \frac{\partial u}{\partial x}=2+4e^{-4y}$ when $x=0$

[S.E.Oct/Nov 2012]

41. A string is stretched tightly between $x=0$ and $x=l$ and both the ends are given the displacement $y=\sin 2t$ perpendicular to the string. If the string satisfies the differential

$\frac{\partial^2 y}{\partial t^2}=4\frac{\partial^2 y}{\partial x^2}$, find the oscillation of the string. [S.E.May/June 2012]

42. Solve: $2\frac{\partial u}{\partial x}-3\frac{\partial u}{\partial y}=0$ when $u(0,y)=3e^{-y}$ [S.E.May/June 2012]

43. Obtain the solution of $\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}$ subject to the condition

(i) $u(0,t)=u(\pi,t)=0$

(ii) $u(x,0)=x$ in every $(0,\pi)$ [S.E.May/June 2012]

44. Solve: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(x,0)=3\sin n\pi x, u(0,t)=0$ for $0<x<1$

[B.tech. May/June 2013]

45. Solve: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ subject to the conditions $y(0,t)=0, y(l,t)=0, \frac{\partial y}{\partial t}=0$ at $t=0$ and $y(x,0)=40\text{x}-x^2, 0 \le x \le 40$ [B.tech. May/June 2013]

46. Solve: $2\text{x}\frac{\partial z}{\partial x} - 3\text{y}\frac{\partial z}{\partial y} = 0$ [B.tech. May/June 2013]

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